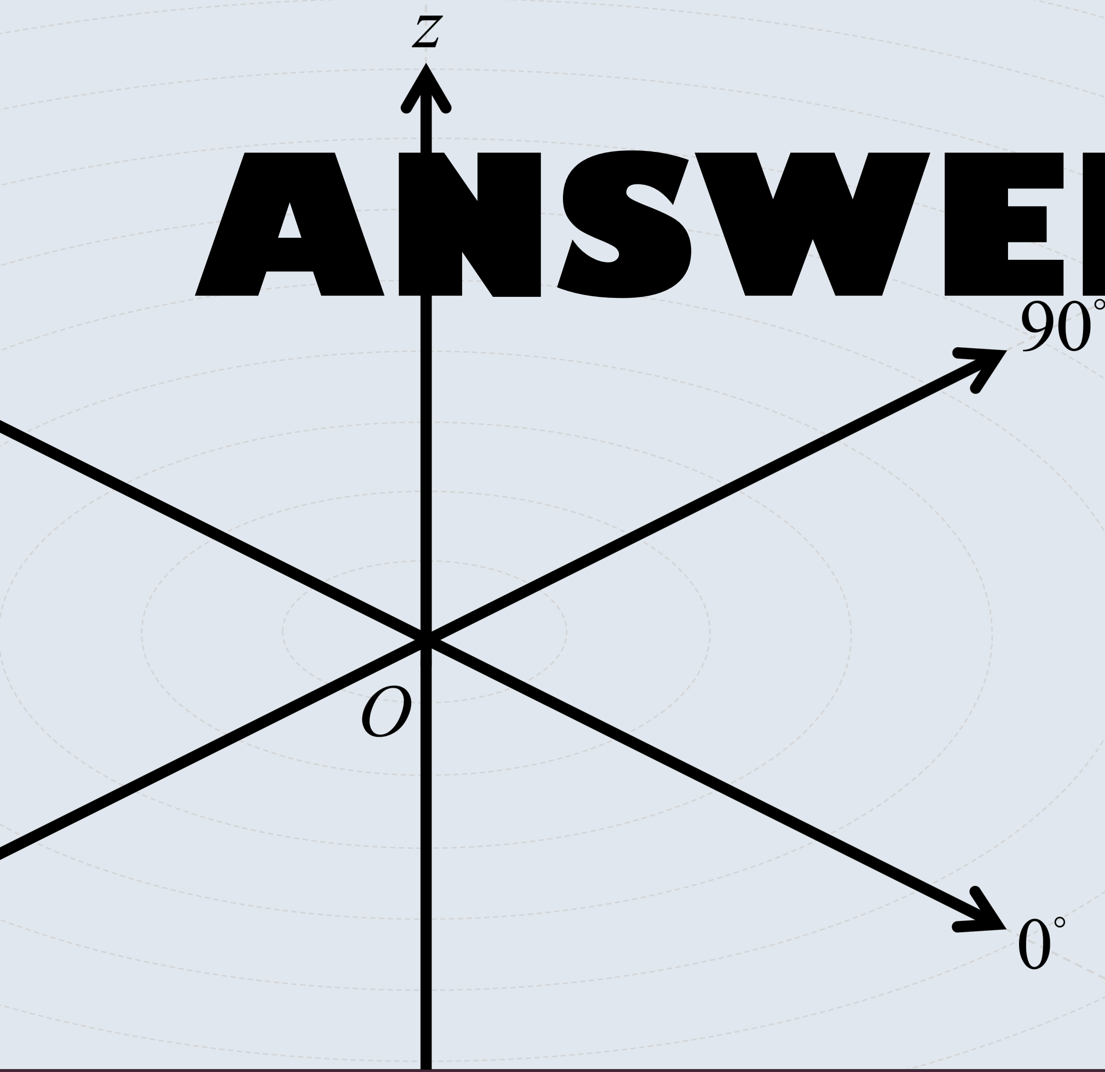
YOUR PRACTICE SET

APPLICATIONS AND INTERPRETATION FOR IBDP MATHEMATICS

FOR HL STUDENTS

Book 2

ANSWERS



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- Compulsory Topics for MAI HL Students
- 80 Example Questions + 320 Intensive Exercise Questions
- Comprehensive Paper 3 Analysis and Practice Questions
- Holistic Exploration on Assessment-styled Questions

SE PRODUCTION LIMITED

Chapter 1 Solution

Exercise 1

1. (a) $y = 8x - 1$

$\Rightarrow x = 8y - 1$ (M1) for swapping variables

$8y = x + 1$

$y = \dfrac{x+1}{8}$ (A1) for changing subject

$\therefore f^{-1}(x) = \dfrac{x+1}{8}$ A1

[3]

(b) (i) 20 A1

(ii) $(f \circ g)(5) = f(20)$

$(f \circ g)(5) = 8(20) - 1$ (A1) for substitution

$(f \circ g)(5) = 159$ A1

[3]

2. (a) $y = 2x - 3$

$\Rightarrow x = 2y - 3$ (M1) for swapping variables

$2y = x + 3$

$y = \dfrac{x+3}{2}$ (A1) for changing subject

$\therefore f^{-1}(x) = \dfrac{x+3}{2}$ A1

[3]

(b) (i) –7 A1

(ii) $(g \circ f)(-2) = g(-7)$

$(g \circ f)(-2) = (-7+5)^2$ (A1) for substitution

$(g \circ f)(-2) = 4$ A1

[3]

3. (a) $y=\sqrt{x+4}$

$\Rightarrow x=\sqrt{y+4}$ (M1) for swapping variables

$4=\sqrt{y+4}$

$16=y+4$

$y=12$ (M1) for valid approach

$\therefore f^{-1}(4)=12$ A1

[3]

(b) (i) 96 A1

(ii) $(f\circ g^{-1})(7)=f(96)$

$(f\circ g^{-1})(7)=\sqrt{96+4}$ (A1) for substitution

$(f\circ g^{-1})(7)=10$ A1

[3]

4. (a) $y=\sqrt{2x-1}$

$\Rightarrow x=\sqrt{2y-1}$ (M1) for swapping variables

$3=\sqrt{2y-1}$

$9=2y-1$

$y=5$ (M1) for valid approach

$\therefore f^{-1}(3)=5$ A1

[3]

(b) $g\left(\dfrac{3a+1}{2}\right)=2$

$\Rightarrow g^{-1}(2)=\dfrac{3a+1}{2}$ (M1) for valid approach

$(f\circ g^{-1})(2)$

$=f\left(\dfrac{3a+1}{2}\right)$

$=\sqrt{2\left(\dfrac{3a+1}{2}\right)-1}$ (A1) for substitution

$=\sqrt{3a}$ A1

[3]

Exercise 2

1. (a) $\{y : y \leq 10\}$ A1

[1]

(b) -7 A1

[1]

(c) $y = -(x+7)^2 + 10$

$\Rightarrow x = -(y+7)^2 + 10$ (M1) for swapping variables

$(y+7)^2 = 10 - x$

$y + 7 = \sqrt{10 - x}$

$y = \sqrt{10 - x} - 7$ (M1) for valid approach

$\therefore f^{-1}(x) = \sqrt{10 - x} - 7$ A1

[3]

2. (a) $f(x) = x^2 - 8x + 40$

$f(x) = x^2 - 8x + 16 + 24$ (M1) for valid approach

$f(x) = (x-4)^2 + 24$ A1

[2]

(b) 4 A1

[1]

(c) $y = (x-4)^2 + 24$

$\Rightarrow x = (y-4)^2 + 24$ (M1) for swapping variables

$(y-4)^2 = x - 24$

$y - 4 = \sqrt{x - 24}$

$y = \sqrt{x - 24} + 4$ (M1) for valid approach

$\therefore f^{-1}(x) = \sqrt{x - 24} + 4$ A1

[3]

3. (a) $f(-14)=4$

$4(-14+h)^2=4$ (M1) for setting equation

$(-14+h)^2=1$

$-14+h=-1$ or $-14+h=1$

$h=13$ or $h=15$ (*Rejected*) A1

[2]

(b) -13 A1

[1]

(c) $y=4(x+13)^2$

$\Rightarrow x=4(y+13)^2$ (M1) for swapping variables

$(y+13)^2=\frac{x}{4}$

$y+13=\sqrt{\frac{x}{4}}$

$y=\sqrt{\frac{x}{4}}-13$ (M1) for valid approach

$\therefore f^{-1}(x)=\sqrt{\frac{x}{4}}-13$ A1

[3]

4. (a) $(1, 0)$ A1

[1]

(b) 1 A1

[1]

(c) $y=((x-3)^2-4)^2$

$\Rightarrow x=((y-3)^2-4)^2$ (M1) for swapping variables

$\sqrt{x}=(y-3)^2-4$

$(y-3)^2=\sqrt{x}+4$

$y-3=\sqrt{\sqrt{x}+4}$

$y=\sqrt{\sqrt{x}+4}+3$ (M1) for valid approach

$\therefore f^{-1}(x)=\sqrt{\sqrt{x}+4}+3$ A1

[3]

Exercise 3

1.	(a)	$g(x) = -f(x)$	(M1) for valid approach
		$g(x) = -(-(x-3)^2 - 1)$	
		$g(x) = (x-3)^2 + 1$	A1
			[2]
	(b) (i)	$3 + p = 1$	(M1) for translation
		$p = -2$	A1
	(ii)	$1 + q = -5$	(M1) for translation
		$q = -6$	A1
			[4]
2.	(a)	$g(x) = f(-x)$	(M1) for valid approach
		$g(x) = (-x-8)^2 - 6$	
		$g(x) = (x+8)^2 - 6$	A1
			[2]
	(b) (i)	$-8 + p = -8$	(M1) for translation
		$p = 0$	A1
	(ii)	$-6 + q = 200$	(M1) for translation
		$q = 206$	A1
			[4]
3.	(a)	$g(x) = f(x+1) - 3$	(M1) for valid approach
		$g(x) = -(x+1-1)^2 + 4 - 3$	(A1) for correct approach
		$g(x) = -x^2 + 1$	A1
			[3]
	(b)	$h(x) = rg(x)$	(M1) for vertical stretch
		$-3x^2 + 3 = r(-x^2 + 1)$	
		$-3x^2 + 3 = -rx^2 + r$	
		$r = 3$	A1
			[2]

4.	(a)	$g(x)=5f(x)$		(M1) for valid approach
		$g(x)=5(x^2+4x+6)$		
		$g(x)=5x^2+20x+30$		A1
				[2]
	(b)	$(-2,10)$		A2
				[2]
	(c)	(i)	$px=3x$	(M1) for horizontal stretch
			$p=3$	A1
		(ii)	$10+q=-2$	(M1) for translation
			$q=-12$	A1
				[4]

Exercise 4

1. (a) $f(-3) = -2$ — (M1) for valid approach

$\therefore f^{-1}(-2) = -3$ — A1

[2]

(b) $f(5) = 3$ — (M1) for valid approach

$(f \circ f)(5) = f(3)$ — (A1) for composite function

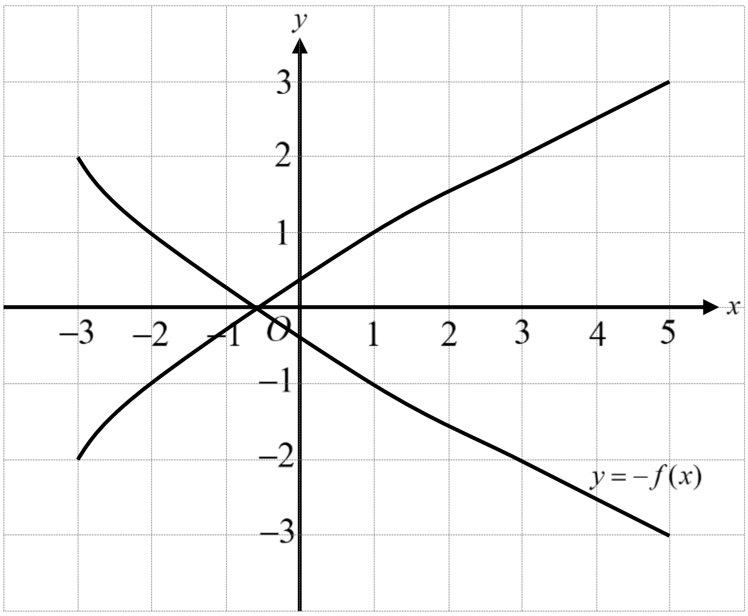
$(f \circ f)(5) = 2$ — A1

[3]

(c) For correct y-intercept — A1

For any two correct points from $(-3, 2)$, $(3, -2)$ and $(5, -3)$ — A1

[2]



2. (a) $f(0)=2$ — (M1) for valid approach

$\therefore f^{-1}(2)=0$ — A1

[2]

(b) $f(4)=0$ — (M1) for valid approach

$(f\circ f)(4)=f(0)$ — (A1) for composite function

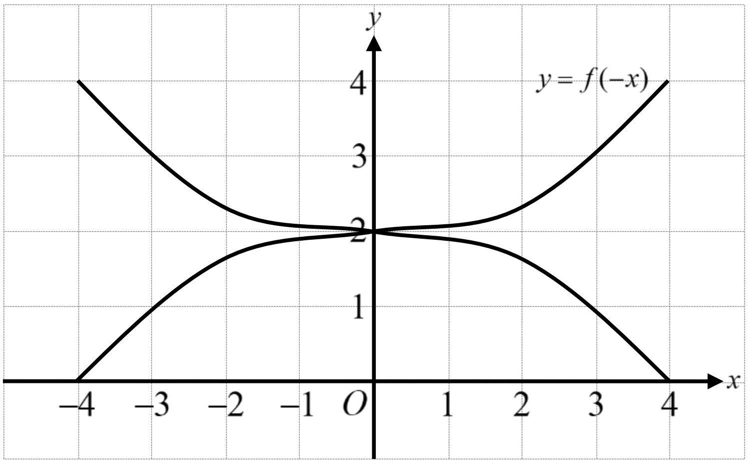
$(f\circ f)(4)=2$ — A1

[3]

(c) For correct y-intercept — A1

For correct points from $(4, 4)$, $(0, 2)$ and $(-4, 0)$ — A1

[2]



3. (a) $-2 \le x \le 3$ A2

[2]

(b) $f^{-1}(1) = -2$ (M1) for valid approach

$(f^{-1} \circ f^{-1})(1) = f^{-1}(-2)$ (A1) for composite function

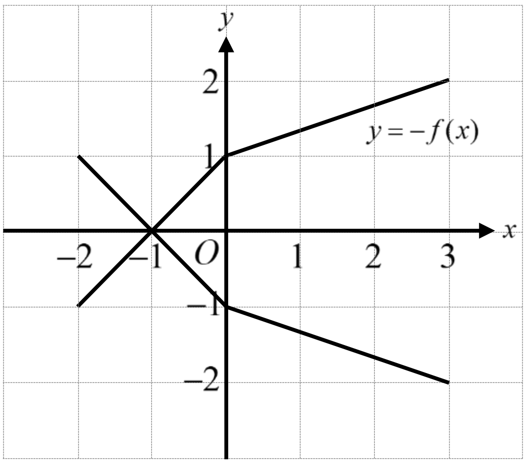
$(f^{-1} \circ f^{-1})(1) = 3$ A1

[3]

(c) For correct y-intercept A1

For any two correct points from $(-2, -1)$, $(0, 1)$ and $(3, 2)$ A1

[2]



4. (a) $-4 \le x \le 3$ A2

[2]

(b) $f^{-1}(3) = -4$ (M1) for valid approach

$(f^{-1} \circ f^{-1})(3) = f^{-1}(-4)$ (A1) for composite function

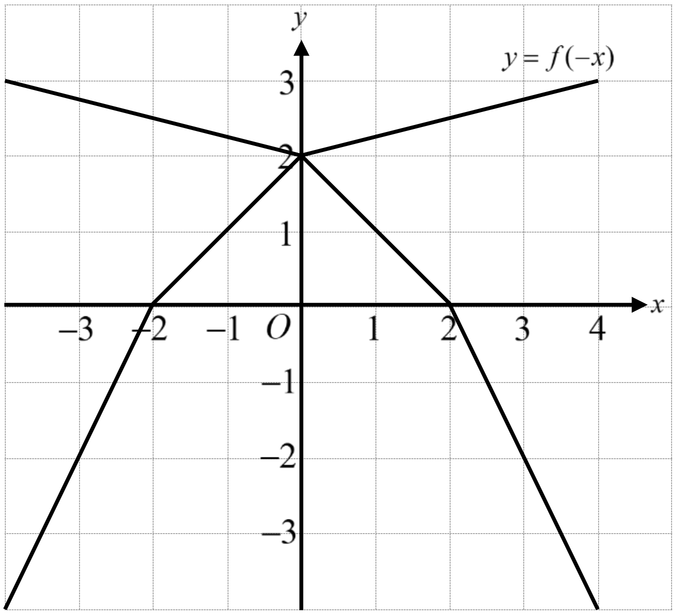
$(f^{-1} \circ f^{-1})(3) = 4$ A1

[3]

(c) For correct y-intercept A1

For correct points $(4, 3)$ and $(-4, -4)$ A1

[2]



Exercise 5

1.	(a)	$C = ks$, where $k \neq 0$	(M1) for valid approach	
		$150 = 12k$		
		$k = 12.5$		
		$\therefore C = 12.5s$	A1	
				[2]
	(b)	\$325	A1	
				[1]
	(c)	The required production cost		
		$= 12.5(2\pi(5))$	(M1) for valid approach	
		$= \$392.6990817$		
		$= \$393$	A1	
				[2]
	(d)	$r = 0.4s^{\frac{1}{3}}$	A1	
				[1]
2.	(a)	$P = \frac{k}{A}$, where $k \neq 0$	(M1) for valid approach	
		$20 = \frac{k}{9}$		
		$k = 180$		
		$\therefore P = \frac{180}{A}$	A1	
				[2]
	(b)	\$2.4	A1	
				[1]
	(c)	The price of a tetrahedron model of a large surface area will approach \$0.	R1	
				[1]
	(d)	-1	A1	
				[1]

3.	(a)	$W = k\sqrt{l}$, where $k \neq 0$	(M1) for valid approach	
		$288 = k\sqrt{256}$		
		$k = 18$		
		$\therefore W = 18\sqrt{l}$	A1	
				[2]
	(b)	441 cm	A1	
				[1]
	(c)	Translate to the left by 3 units	A1	
		Followed by a vertical stretch of scale factor 2	A1	
				[2]
4.	(a)	$f(x) = \frac{k}{x^2}$, where $k \neq 0$	(M1) for valid approach	
		$0.375 = \frac{k}{4^2}$		
		$k = 6$		
		$\therefore f(x) = \frac{6}{x^2}$	A1	
				[2]
	(b)	$x = 0$	A1	
				[1]
	(c)	$g(x) = 4f(x-3)$	(M1) for valid approach	
		$g(x) = 4\left(\frac{6}{(x-3)^2}\right)$		
		$g(x) = \frac{24}{(x-3)^2}$	A1	
				[2]

Chapter 2 Solution

Exercise 6

1. (a)

$\log\frac{1}{25}+\log\frac{1}{40}=\log\left(\frac{1}{25}\cdot\frac{1}{40}\right)$ (A1) for correct formula

$\log\frac{1}{25}+\log\frac{1}{40}=\log\frac{1}{1000}$

$\log\frac{1}{25}+\log\frac{1}{40}=\log 10^{-3}$ (A1) for valid approach

$\log\frac{1}{25}+\log\frac{1}{40}=-3$ A1

[3]

(b)

$\ln e^{7.5}-\ln e\sqrt{e}=\ln\frac{e^{7.5}}{e\sqrt{e}}$ (A1) for correct formula

$\ln e^{7.5}-\ln e\sqrt{e}=\ln e^{7.5-1.5}$

$\ln e^{7.5}-\ln e\sqrt{e}=\ln e^{6}$ (A1) for valid approach

$\ln e^{7.5}-\ln e\sqrt{e}=6$ A1

[3]

2. (a)

$\log 0.8+\log 1250=\log(0.8\cdot 1250)$ (A1) for correct formula

$\log 0.8+\log 1250=\log 1000$

$\log 0.8+\log 1250=\log 10^{3}$ (A1) for valid approach

$\log 0.8+\log 1250=3$ A1

[3]

(b)

$\ln\sqrt[3]{e}-\ln e^{\frac{4}{3}}=\ln\frac{\sqrt[3]{e}}{e^{\frac{4}{3}}}$ (A1) for correct formula

$\ln\sqrt[3]{e}-\ln e^{\frac{4}{3}}=\ln e^{\frac{1}{3}-\frac{4}{3}}$

$\ln\sqrt[3]{e}-\ln e^{\frac{4}{3}}=\ln e^{-1}$ (A1) for valid approach

$\ln\sqrt[3]{e}-\ln e^{\frac{4}{3}}=-1$ A1

[3]

3. (a) $\log 112 + \log \frac{25}{4} - \log 7 = \log \frac{112 \cdot \frac{25}{4}}{7}$ (A1) for correct formula

$\log 112 + \log \frac{25}{4} - \log 7 = \log 100$

$\log 112 + \log \frac{25}{4} - \log 7 = \log 10^2$ (A1) for valid approach

$\log 112 + \log \frac{25}{4} - \log 7 = 2$ A1

[3]

(b) $e^{\ln\sqrt{x}} = 3$

$\ln\sqrt{x} = \ln 3$ (M1) for valid approach

$\therefore \sqrt{x} = 3$ (A1) for valid approach

$x = 9$ A1

[3]

4. (a) $\ln \frac{1}{3} + \ln 45 - \ln 15 = \ln \frac{\frac{1}{3} \cdot 45}{15}$ (A1) for correct formula

$\ln \frac{1}{3} + \ln 45 - \ln 15 = \ln 1$

$\ln \frac{1}{3} + \ln 45 - \ln 15 = \ln e^0$ (A1) for valid approach

$\ln \frac{1}{3} + \ln 45 - \ln 15 = 0$ A1

[3]

(b) $10^{\log x^3} = 1331$

$\log x^3 = \log 1331$ (M1) for valid approach

$\therefore x^3 = 1331$ (A1) for valid approach

$x = 11$ A1

[3]

Exercise 7

1. (a) $y = \log \sqrt[3]{x}$

$\Rightarrow x = \log \sqrt[3]{y}$ — (M1) for swapping variables

$\sqrt[3]{y} = 10^x$

$y = (10^x)^3$ — (A1) for changing subject

$\therefore f^{-1}(x) = 10^{3x}$ — A1

[3]

(b) $\{y : y > 0\}$ — A1

[1]

(c) $g(\sqrt{10}) = \log(\sqrt{10})^4$ — (M1) for substitution

$g(\sqrt{10}) = 2$ — (A1) for correct value

$(f^{-1} \circ g)(\sqrt{10}) = f^{-1}(g(\sqrt{10}))$

$(f^{-1} \circ g)(\sqrt{10}) = f^{-1}(2)$

$(f^{-1} \circ g)(\sqrt{10}) = 10^{3(2)}$ — (M1) for substitution

$(f^{-1} \circ g)(\sqrt{10}) = 1000000$ — A1

[4]

2. (a) $y = e^{4x}$

$\Rightarrow x = e^{4y}$ — (M1) for swapping variables

$4y = \ln x$

$y = 0.25 \ln x$ — (A1) for changing subject

$\therefore f^{-1}(x) = 0.25 \ln x$ — A1

[3]

(b) $\{x : x > 0\}$ — A1

[1]

(c) $f^{-1}(16) = 0.25 \ln 16$ — (M1) for substitution

$f^{-1}(16) = \ln 2$ — (A1) for correct value

$(g \circ f^{-1})(16) = g(f^{-1}(16))$

$(g \circ f^{-1})(16) = g(\ln 2)$

$(g \circ f^{-1})(16) = (e^{\ln 2} - 1)^3$ — (M1) for substitution

$(g \circ f^{-1})(16) = 1$ — A1

[4]

3. (a) $y = \ln x + 3$

$\Rightarrow x = \ln y + 3$ (M1) for swapping variables

$x - 3 = \ln y$

$y = e^{x-3}$ (A1) for changing subject

$\therefore f^{-1}(x) = e^{x-3}$ A1

[3]

(b) $\{y : y > 0\}$ A1

[1]

(c) $g(2) = e^{(2+1)(2-3)}$ (M1) for substitution

$g(2) = e^{-3}$ (A1) for correct value

$(f \circ g)(2) = f(g(2))$

$(f \circ g)(2) = f(e^{-3})$

$(f \circ g)(2) = \ln e^{-3} + 3$ (M1) for substitution

$(f \circ g)(2) = 0$ A1

[4]

4. (a) $y = 10^{3x}$

$\Rightarrow x = 10^{3y}$ (M1) for swapping variables

$3y = \log x$

$y = \frac{1}{3}\log x$ (A1) for changing subject

$\therefore f^{-1}(x) = \frac{1}{3}\log x$ A1

[3]

(b) $\{y : y \in \mathbb{R}\}$ A1

[1]

(c) $(g \circ f)(x) = g(f(x))$

$(g \circ f)(x) = (1 + \log f(x))^2$

$(g \circ f)(x) = (1 + \log 10^{3x})^2$ (M1) for substitution

$(g \circ f)(x) = (1 + 3x)^2$ (A1) for correct approach

$(g \circ f)(x) = 9x^2 + 6x + 1$ A1

[3]

Exercise 8

1. (a) (i) The required amount of bacteria

$= B(0)$

$= \frac{12}{1+5e^{-0.06(0)}}$ (M1) for substitution

$= 2$ million A1

(ii) The required amount of bacteria

$= B(26)$

$= \frac{12}{1+5e^{-0.06(26)}}$ (M1) for substitution

$= 5.851716463$

$= 5.85$ million A1

[4]

(b) $\{B : 2 \le B < 12\}$ A2

[2]

2. (a) The required weight of the substance

$= W(0)$

$= \frac{27}{1+8e^{-0.11(0)}}$ (M1) for substitution

$= 3$ g A1

[2]

(b) $W(t) = 20$

$\frac{27}{1+8e^{-0.11t}} = 20$ (M1) for setting equation

$\frac{27}{1+8e^{-0.11t}} - 20 = 0$

By considering the graph of $y = \frac{27}{1+8e^{-0.11t}} - 20$,

$t = 28.447852$.

Thus, the required date is 30th July. A1

[2]

(c) $W = 27$ A1

[1]

3. (a) $P(0)=100000$

$\frac{600000}{1+Ce^{-0.11(0)}}=100000$ (M1) for setting equation

$\frac{600000}{1+C}=100000$

$1+C=6$

$C=5$ A1

[2]

(b) $P(t)=450000$

$\frac{600000}{1+5e^{-0.11t}}=450000$ (M1) for setting equation

$\frac{600000}{1+5e^{-0.11t}}-450000=0$

By considering the graph of

$y=\frac{600000}{1+5e^{-0.11t}}-450000$, $t=24.618638$.

$\therefore t=24.6$ A1

[2]

(c) $P=600000$ A1

[1]

4. (a) The value of the vase at the end of 1907

$=V(7)$

$=\frac{250000}{1+19e^{-1.6}}(7+10)$ (M1) for substitution

$=\$878819.3257$

$=\$879000$ A1

[2]

(b) $V(t)=2000000$

$\frac{5000000}{1+19e^{-0.16t}}=2000000$ (M1) for setting equation

$\frac{5000000}{1+19e^{-0.16t}}-2000000=0$

By considering the graph of

$y=\frac{5000000}{1+19e^{-0.16t}}-2000000$, $t=15.868587$.

$\therefore t=15.9$ A1

[2]

(c) The value of the vase will approach $5000000 after a long period of time. R1

[1]

Exercise 9

1. (a) $T = hk^x$

$\ln T = \ln(hk^x)$ (A1) for correct approach

$\ln T = \ln h + \ln k^x$ (A1) for correct approach

$\ln T = (\ln k)x + \ln h$ A1

[3]

(b) (i) $\ln h = -1.7$

$h = e^{-1.7}$ (M1) for valid approach

$h = 0.182683524$

$h = 0.1827$ A1

(ii) $\ln k = 0.06$

$k = e^{0.06}$ (M1) for valid approach

$k = 1.061836547$

$k = 1.0618$ A1

[4]

2. (a) $W = hx^k$

$\ln W = \ln(hx^k)$ (A1) for correct approach

$\ln W = \ln h + \ln x^k$ (A1) for correct approach

$\ln W = k \ln x + \ln h$ A1

[3]

(b) $k = \dfrac{1.5 - 0}{0 - 0.3}$ (M1) for valid approach

$k = -5$ A1

[2]

(c) $\ln h = 1.5$

$h = e^{1.5}$ (M1) for valid approach

$h = 4.48168907$

$h = 4.4817$ A1

[2]

3.	(a)	$\dfrac{b-0}{0-(-6)}=3$	(M1) for valid approach
		$\dfrac{b}{6}=3$	
		$b=18$	A1
			[2]
	(b)	$\ln V=3x+18$	
		$V=e^{3x+18}$	A1
			[1]
	(c)	The difference of V	
		$=\dfrac{e^{3(1)+18}}{e^{3(0.5)+18}}$	M1A1
		$=4.48168907$	
		$=4.48$	A1
			[3]
4.	(a)	(i) $a=\dfrac{1-0}{2.5-5}$	(M1) for valid approach
		$a=-0.4$	A1
		(ii) $-0.4=\dfrac{b-0}{0-5}$	(M1) for valid approach
		$b=2$	A1
			[4]
	(b)	$\ln N=-0.4\ln t+2$	
		$\ln N=\ln t^{-0.4}+\ln e^{2}$	(A1) for correct approach
		$\ln N=\ln(e^{2}t^{-0.4})$	(A1) for correct approach
		$N=e^{2}t^{-0.4}$	A1
			[3]

Exercise 10

1. (a) Initial number

$= 2500e^{0.075(0)}$ (A1) for substitution

$= 2500$ A1

[2]

(b) The required number

$= 2500e^{0.075(10)}$ (A1) for substitution

$= 5292.500042$

$= 5290$ A1

[2]

(c) $8000 = 2500e^{0.075t}$ (M1) for setting equation

$e^{0.075t} = 3.2$

$0.075t = \ln 3.2$ (A1) for correct approach

$t = \frac{1}{0.075}\ln 3.2$

$t = 15.5$

Thus, it takes 15.5 years to reach 8000 leopards. A1

[3]

(d) $B(10) = 5000$ (M1) for setting equation

$ke^{\frac{1800}{k}} = 5000$

$ke^{\frac{1800}{k}} - 5000 = 0$ (A1) for correct approach

By considering the graph of $y = ke^{\frac{1800}{k}} - 5000$,

$k = 1472.0674$.

$\therefore k = 1470$ A1

[3]

(e) $B(t) > A(t)$ (M1) for setting inequality

$B(t) - A(t) > 0$

$1472.0674e^{\frac{180}{1472.0674}t} - 2500e^{0.075t} > 0$ A1

By considering the graph of

$y = 1472.0674e^{\frac{180}{1472.0674}t} - 2500e^{0.075t}$, $t > 11.202547$. (A1) for correct approach

$\therefore n = 12$ A1

[4]

2. (a) Initial number

$= 420 \times 1.15^0$ (A1) for substitution

$= 420$ A1

[2]

(b) The required number

$= 420 \times 1.15^6$ (A1) for substitution

$= 971.4855216$

$= 971$ A1

[2]

(c) $420 \times 1.15^t = 750$ (M1) for setting equation

$420 \times 1.15^t - 750 = 0$ (A1) for correct approach

By considering the graph of $y = 420 \times 1.15^t - 750$,

$t = 4.148615$.

Thus, it takes 4.15 years to reach 750 trams. A1

[3]

(d) $\dfrac{4680000}{70e^{-5k} + 130} = 27500$ (M1) for setting equation

$70e^{-5k} + 130 = \dfrac{1872}{11}$

$70e^{-5k} = \dfrac{442}{11}$

$e^{-5k} = \dfrac{221}{385}$ (M1) for valid approach

$-5k = \ln \dfrac{221}{385}$ (A1) for correct approach

$k = 0.1110161266$

$k = 0.111$ A1

[4]

(e) $420 \times 1.15^n > 5\left(\dfrac{4680000}{70e^{-0.1110161266n} + 130}\right)$ M1A1

$420 \times 1.15^n - \dfrac{23400000}{70e^{-0.1110161266n} + 130} > 0$ (A1) for correct approach

By considering the graph of

$y = 420 \times 1.15^n - \dfrac{23400000}{70e^{-0.1110161266n} + 130}$,

$n > 43.331409$.

$\therefore n = 44$ A1

[4]

3. (a) Initial number

$=1050\times1.25^{0}$ (A1) for substitution

$=1050$ A1

[2]

(b) The required number

$=1050\times1.25^{16}$ (A1) for substitution

$=37303.49363$

$=37300$ A1

[2]

(c) $1050\times1.25^{t}=4200$ (M1) for setting equation

$1.25^{t}=4$

$\ln1.25^{t}=\ln4$ (A1) for correct approach

$t\ln1.25=\ln4$ (A1) for correct approach

$t=\dfrac{\ln4}{\ln1.25}$

$t=6.212567439$

Thus, it takes 6.21 weeks to reach 4200 cars. A1

[4]

(d) $\dfrac{410000}{95e^{-12k}+75k}=4600$ (M1) for setting equation

$\dfrac{410000}{95e^{-12k}+75k}-4600=0$ (A1) for correct approach

By considering the graph of

$y=\dfrac{410000}{95e^{-12k}+75k}-4600$, $k=1.188405$.

$\therefore k=1.19$ A1

[3]

(e) $1050\times1.25^{n}>2\left(\dfrac{410000}{95e^{-1.188405n}+75(1.188405)}\right)$ M1A1

$1050\times1.25^{n}-\dfrac{820000}{95e^{-1.188405n}+75(1.188405)}>0$ (A1) for correct approach

By considering the graph of

$y=1050\times1.25^{n}-\dfrac{820000}{95e^{-1.188405n}+75(1.188405)}$,

$n>9.7264913$.

$\therefore n=10$ A1

[4]

4. (a) Initial pressure

$= 4 \times e^{0.12(30)}$ (A1) for substitution

$= 146.3929378$

$= 146$ A1

[2]

(b) $4e^{0.12t} = 8$ (M1) for setting equation

$e^{0.12t} = 2$

$0.12t = \ln 2$ (A1) for correct approach

$t = 5.776226505$

Hence it takes 5.78 minutes to reach 8 units. A1

[3]

(c) (i) $Q(0) = 3.5$

$Q_0 e^{k(0)} = 3.5$ (M1) for setting equation

$Q_0 = 3.5$ A1

(ii) $Q(30) = 171$

$3.5e^{k(30)} = 171$ (M1) for setting equation

$e^{30k} = 48.85714286$

$30k = \ln 48.85714286$ (A1) for correct approach

$k = 0.1296300196$

$k = 0.130$ A1

[5]

(d) $4e^{0.12n} + 3.5e^{0.1296300196n} > 400$ M1A1

$4e^{0.12n} + 3.5e^{0.1296300196n} - 400 > 0$ (A1) for correct approach

By considering the graph of

$y = 4e^{0.12n} + 3.5e^{0.1296300196n} - 400$,

$n > 31.847494$

$\therefore n = 32$ A1

[4]

Chapter 3 Solution

Exercise 11

1. (a) $r = \dfrac{-324}{-540}$ (M1) for valid approach

$r = 0.6$ A1

[2]

(b) $S_{10} = \dfrac{u_1(1-r^{10})}{1-r}$

$S_{10} = \dfrac{-900(1-0.6^{10})}{1-0.6}$ (A1) for substitution

$S_{10} = -2236.39511$

$S_{10} = -2240$ A1

[2]

(c) $S_{\infty} = \dfrac{u_1}{1-r}$

$S_{\infty} = \dfrac{-900}{1-0.6}$ (A1) for substitution

$S_{\infty} = -2250$ A1

[2]

2. (a) $r = \dfrac{\ln x^{24}}{\ln x^{48}}$ (M1) for valid approach

$r = \dfrac{24\ln x}{48\ln x}$ (A1) for correct approach

$r = \dfrac{1}{2}$ A1

[3]

(b) $u_6 = u_1 \times r^{6-1}$

$u_6 = 48\ln x \times \left(\dfrac{1}{2}\right)^{6-1}$ (A1) for substitution

$u_6 = \dfrac{3}{2}\ln x$ A1

[2]

(c) $S_\infty = \dfrac{u_1}{1-r}$

$S_\infty = \dfrac{48\ln x}{1-\dfrac{1}{2}}$ (A1) for substitution

$S_\infty = 96\ln x$ A1

[2]

3. (a) $r = \frac{e^{8x}}{e^{12x}}$ (M1) for valid approach

$r = e^{-4x}$ A1

[2]

(b) $u_7 = u_1 \times r^{7-1}$

$u_7 = e^{12x} \times (e^{-4x})^{7-1}$ (A1) for substitution

$u_7 = e^{-12x}$ A1

[2]

(c) $S_\infty = \frac{u_1}{1-r}$

$S_\infty = \frac{e^{12x}}{1-e^{-4x}}$

$S_\infty = \frac{e^{16x}}{e^{4x}-1}$ (A1) for correct working

$\frac{e^{16x}}{e^{4x}-1} = \frac{e^{96}}{e^{24}-1}$ (M1) for setting equation

$e^{16x} = e^{96}$

$16x = 96$

$x = 6$ A1

[3]

4. (a) $r = \dfrac{3^{9x}}{3^{10x}}$ (M1) for valid approach

$r = 3^{-x}$ A1

[2]

(b) $u_n = u_1 \times r^{n-1}$

$u_n = 3^{10x} \times (3^{-x})^{n-1}$ (A1) for substitution

$u_n = 3^{10x} \times 3^{-nx+x}$ (A1) for correct approach

$u_n = 3^{(11-n)x}$ A1

[3]

(c) $3^{-x} = \dfrac{1}{3}$ (M1) for setting equation

$3^{-x} = 3^{-1}$

$x = 1$

$S_\infty = \dfrac{u_1}{1-r}$

$S_\infty = \dfrac{3^{10}}{1-3^{-1}}$ (A1) for substitution

$S_\infty = \dfrac{1}{2} \times 3^{11}$ A1

[3]

Exercise 12

1.	(a)	$r = \frac{1}{2}$ leads to a finite sum.	A1
		As $-1 < \frac{1}{2} < 1$.	R1
			[2]
	(b)	$u_1 = \frac{10}{\frac{1}{2}}$	(A1) for substitution
		$u_1 = 20$	(A1) for correct value
		$S_\infty = \frac{u_1}{1-r}$	
		$S_\infty = \frac{20}{1-\frac{1}{2}}$	(A1) for substitution
		$S_\infty = 40$	A1
			[4]
2.	(a)	$r = -\frac{1}{3}$ leads to a finite sum.	A1
		As $-1 < -\frac{1}{3} < 1$.	R1
			[2]
	(b)	$r = 3$	
		$u_1 = \frac{27}{3^2}$	(A1) for substitution
		$u_1 = 3$	(A1) for correct value
		$S_4 = \frac{u_1(1-r^4)}{1-r}$	
		$S_4 = \frac{3(1-3^4)}{1-3}$	(A1) for substitution
		$S_4 = 120$	A1
			[4]

3. (a) $r = \frac{1}{2}$

$u_1 = \log_2 x^{\frac{1}{2}}$ (A1) for correct value

$S_\infty = \frac{\log_2 x^{\frac{1}{2}}}{1 - \frac{1}{2}}$ (A1) for substitution

$S_\infty = 2\log_2 x^{\frac{1}{2}}$

$S_\infty = \log_2 x$ A1

[3]

(b) $r = -2$

$u_1 = \log_2 \left(\frac{1}{2}\right)^{-2}$ (A1) for substitution

$u_1 = 2$ (A1) for correct value

$S_6 = \frac{2\left(1 - \left(\frac{1}{2}\right)^6\right)}{1 - \frac{1}{2}}$ (A1) for substitution

$S_6 = \frac{63}{16}$ A1

[4]

4. (a) $r = -\frac{1}{3}$

$\frac{9}{m+2} = -\frac{1}{3}$ (M1) for setting equation

$-27 = m + 2$ (A1) for correct approach

$m = -29$ A1

[3]

(b) $u_1 = \frac{-29+2}{-\frac{1}{3}}$ (A1) for substitution

$u_1 = 81$ (A1) for correct value

$S_\infty = \frac{u_1}{1-r}$

$S_\infty = \frac{81}{1-\left(-\frac{1}{3}\right)}$ (A1) for substitution

$S_\infty = \frac{243}{4}$ A1

[4]

Chapter 4 Solution

Exercise 13

1. (a) $\cos\theta = -\sqrt{1-\sin^2\theta}$ (A1) for correct approach

$\cos\theta = -\sqrt{1-\left(\frac{5}{13}\right)^2}$ (A1) for substitution

$\cos\theta = -\frac{12}{13}$ A1

[3]

(b) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{\frac{5}{13}}{-\frac{12}{13}}$ (A1) for substitution

$\tan\theta = -\frac{5}{12}$ A1

[2]

2. (a) $\cos\theta = \frac{5}{\sqrt{(\sqrt{11})^2+5^2}}$ M1

$\cos\theta = \frac{5}{\sqrt{36}}$ A1

$\cos\theta = \frac{5}{6}$ AG

[2]

(b) $\sin\theta = -\sqrt{1-\cos^2\theta}$ (A1) for correct approach

$\sin\theta = -\sqrt{1-\left(\frac{5}{6}\right)^2}$ (A1) for substitution

$\sin\theta = -0.552770798$

$\sin\theta = -0.553$ A1

[3]

3.	(a)	(i)	0.64	A1
		(ii)	$\sin\theta = -\sqrt{1-\cos^2\theta}$	(A1) for correct approach
			$\sin\theta = -\sqrt{1-0.64}$	(A1) for substitution
			$\sin\theta = -0.6$	A1
				[4]
	(b)		$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	
			$\tan\theta = \dfrac{-0.6}{-\sqrt{0.64}}$	(A1) for substitution
			$\tan\theta = 0.75$	A1
				[2]
4.	(a)	(i)	$-\dfrac{9}{40}$	A1
		(ii)	$\sin\theta = \dfrac{9}{\sqrt{9^2+40^2}}$	(M1) for valid approach
			$\sin\theta = \dfrac{9}{\sqrt{1681}}$	
			$\sin\theta = \dfrac{9}{41}$	A1
				[3]
	(b)		$\cos\theta = -\sqrt{1-\sin^2\theta}$	(A1) for correct approach
			$\cos\theta = -\sqrt{1-\left(\dfrac{9}{41}\right)^2}$	(A1) for substitution
			$\cos\theta = -\dfrac{40}{41}$	A1
				[3]

Exercise 14

1. (a) $5-2\cos^2 x=3\cos x$

$5-2\cos^2 x-3\cos x=0$ (M1) for valid approach

By considering the graph of $y=5-2\cos^2 x-3\cos x$, $x=0$, $x=6.2831857$ or $x=12.566371$.

$\therefore x=0$, $x=6.28$ or $x=12.6$ A3

[4]

(b) 4 A1

[1]

2. (a) $\cos 2x=4-7\sin x$

$\cos 2x+7\sin x-4=0$ (M1) for valid approach

By considering the graph of $y=\cos 2x+7\sin x-4$, $x=0.5235988$ or $x=2.6179939$.

$\therefore x=0.524$ or $x=2.62$ A2

[3]

(b) 1 A1

[1]

3. (a) $\sin x-\sin x\cos x=0$

By considering the graph of $y=\sin x-\sin x\cos x$, $x=3.1415927$, $x=6.2831853$ or $x=9.424778$.

$\therefore x=3.14$, $x=6.28$ or $x=9.42$ A3

[3]

(b) 7 A1

[1]

4. (a) $\cos 2x-\sin 4x=0$

By considering the graph of $y=\cos 2x-\sin 4x$, $x=0.2617994$, $x=0.7853982$ or $x=1.3089969$.

$\therefore x=0.262$, $x=0.785$ or $x=1.31$ A3

[3]

(b) 4 A1

[1]

Exercise 15

1. (a) $h(x) = g(f(x))$ (M1) for composite function

$h(x) = 3\cos\left(\frac{f(x)}{4}\right) - 5$ (A1) for substitution

$h(x) = 3\cos\left(\frac{2x+3}{4}\right) - 5$

$h(x) = 3\cos\left(\frac{1}{2}x + \frac{3}{4}\right) - 5$ A1

[3]

(b) The period of h

$= 2\pi \div \frac{1}{2}$ (M1) for valid approach

$= 4\pi$ A1

[2]

(c) $\{y : -8 \le y \le -2\}$ A2

[2]

2. (a) $h(x) = g(f(x))$ (M1) for composite function

$h(x) = 4\sin\left(\frac{f(x)}{2}\right) - 3$ (A1) for substitution

$h(x) = 4\sin\left(\frac{8x+7}{2}\right) - 3$

$h(x) = 4\sin\left(4x + \frac{7}{2}\right) - 3$ A1

[3]

(b) The period of h

$= \frac{2\pi}{4}$ (M1) for valid approach

$= \frac{\pi}{2}$ A1

[2]

(c) 4 A1

[1]

3.	(a)	$h(x) = f(g(x))$	(M1) for composite function
		$h(x) = \frac{3}{2}\left(4\sin\left(\frac{x}{3}\right)+13\right)-1$	(A1) for substitution
		$h(x) = 6\sin\frac{1}{3}x+\frac{37}{2}$	A1
			[3]
	(b)	6	A1
			[1]
	(c)	$\left\{y : \frac{25}{2} \le y \le \frac{49}{2}\right\}$	A2
			[2]
4.	(a)	$h(x) = f(g(x))$	(M1) for composite function
		$h(x) = 1-2(6\cos\pi x+1)$	(A1) for substitution
		$h(x) = -12\cos\pi x-1$	A1
			[3]
	(b)	12	A1
			[1]
	(c)	$\{y : -13 \le y \le 11\}$	A2
			[2]

Chapter 5 Solution

Exercise 16

1. (a) The length of arc ABC

$= (32.1)(1.44)$ — (A1) for substitution

$= 46.224$

$= 46.2 \text{ cm}$ — A1

[2]

(b) The perimeter of sector $OABC$

$= 46.224 + 32.1 + 32.1$ — (M1) for valid approach

$= 110.424$

$= 110 \text{ cm}$ — A1

[2]

(c) The area of sector $OABC$

$= \frac{1}{2}(32.1)^2(1.44)$ — (A1) for substitution

$= 741.8952$

$= 742 \text{ cm}^2$ — A1

[2]

2. (a) The length of arc ABC

$= (12)(2\pi - 2.6)$ — (M1)(A1) for substitution

$= 44.19822369$

$= 44.2 \text{ cm}$ — A1

[3]

(b) The perimeter of sector $OABC$

$= 44.19822369 + 12 + 12$ — (M1) for valid approach

$= 68.19822369$

$= 68.2 \text{ cm}$ — A1

[2]

(c) The area of sector $OABC$

$= \frac{1}{2}(12)^2(2\pi - 2.6)$ — (A1) for substitution

$= 265.1893421$

$= 265 \text{ cm}^2$ — A1

[2]

3. (a) $\frac{1}{2}(\text{OC})^2(1.93) = 603$ (A1) for correct equation

$\text{OC}^2 = 624.8704663$

$\text{OC} = 24.99740919$

$\text{OC} = 25.0 \text{ cm}$ A1

[2]

(b) (i) The reflex $\text{A}\hat{\text{O}}\text{C}$

$= (2\pi - 1.93) \text{ rad}$ (M1) for valid approach

$= 4.353185307 \text{ rad}$

$= 4.35 \text{ rad}$ A1

(ii) The area of sector OADC

$= \frac{1}{2}(24.99740919)^2(4.353185307)$ (A1) for substitution

$= 1360.088466$

$= 1360 \text{ cm}^2$ A1

[4]

4. (a) $4.5\theta = 4.32$ (A1) for correct equation

$\theta = 0.96 \text{ rad}$ A1

[2]

(b) (i) The reflex $\text{A}\hat{\text{O}}\text{C}$

$= (2\pi - 0.96) \text{ rad}$ (M1) for valid approach

$= 5.323185307 \text{ rad}$

$= 5.32 \text{ rad}$ A1

(ii) The area of sector OADC

$= \frac{1}{2}(4.5)^2(5.323185307)$ (A1) for substitution

$= 53.89725124$

$= 53.9 \text{ cm}^2$ A1

[4]

Exercise 17

1. (a) The exact length of arc ACB

$= (18)(2.41)$ (A1) for substitution

$= 43.38 \text{ cm}$ A1

[2]

(b) $AB = \sqrt{18^2 + 18^2 - 2(18)(18)\cos 2.41}$ (M1)(A1) for substitution

$AB = 33.6182118$

$AB = 33.6 \text{ cm}$ A1

[3]

(c) The required perimeter

$= 33.6182118 + 43.38$ (M1) for correct approach

$= 76.9982118$

$= 77.0 \text{ cm}$ A1

[2]

2. (a) The area of the sector OACB

$= \frac{1}{2}(2)^2(0.68)$ (A1) for substitution

$= 1.36 \text{ cm}^2$ A1

[2]

(b) The required area

$= \frac{1}{2}(2)(2)\sin 0.68$ (A1) for substitution

$= 1.257586048$

$= 1.26 \text{ cm}^2$ A1

[2]

(c) The required area

$= 1.36 - 1.257586048$ (M1) for valid approach

$= 0.102413952$

$= 0.102 \text{ cm}^2$ A1

[2]

3. (a) $\cos A\hat{O}B = \dfrac{15.6^2 + 15.6^2 - 25^2}{2(15.6)(15.6)}$ (M1)(A1) for substitution

$\cos A\hat{O}B = -0.284105851$

$A\hat{O}B = 1.858870049 \text{ rad}$

$A\hat{O}B = 1.86 \text{ rad}$ A1

[3]

(b) The area of the sector AOB

$= \dfrac{1}{2}(15.6)^2(1.858870049)$ (A1) for substitution

$= 226.1873075$

$= 226 \text{ cm}^2$ A1

[2]

(c) The area of the shaded region

$= 226.1873075 - 117$ (M1) for valid approach

$= 109.1873075$

$= 109 \text{ cm}^2$ A1

[2]

4. (a) $\cos A\hat{O}B = \dfrac{24^2 + 24^2 - 37^2}{2(24)(24)}$ (M1)(A1) for substitution

$\cos A\hat{O}B = -0.188368055$

$A\hat{O}B = 1.760296516 \text{ rad}$

$A\hat{O}B = 1.76 \text{ rad}$ A1

[3]

(b) The length of the minor arc AB

$= (24)(1.760296516)$ (A1) for substitution

$= 42.2471164$

$= 42.2 \text{ cm}$ A1

[2]

(c) The perimeter of the shaded segment

$= (2\pi(24) - 42.2471164) + 37$ (M1)(A1) for substitution

$= 145.549331$

$= 146 \text{ cm}$ A1

[3]

Exercise 18

1. (a) $\frac{\sin A\hat{B}C}{AC}=\frac{\sin B\hat{A}C}{BC}$ (M1) for sine rule

$\frac{\sin A\hat{B}C}{80}=\frac{\sin 20^\circ}{30}$ (A1) for substitution

$\sin A\hat{B}C=\frac{80\sin 20^\circ}{30}$

$A\hat{B}C=65.79072072^\circ$ or $180^\circ-65.79072072^\circ$

$A\hat{B}C=65.8^\circ$ or 114° A2

[4]

(b) $A\hat{C}B+A\hat{B}C+B\hat{A}C=180^\circ$ (M1) for valid approach

$A\hat{C}B+114.2092793^\circ+20^\circ=180^\circ$

$A\hat{C}B=45.79072072^\circ$

$A\hat{C}B=45.8^\circ$ A1

[2]

2. (a) $\frac{\sin B\hat{A}C}{BC}=\frac{\sin A\hat{B}C}{AC}$ (M1) for sine rule

$\frac{\sin B\hat{A}C}{20}=\frac{\sin 33^\circ}{\sqrt{151}}$ (A1) for substitution

$\sin B\hat{A}C=\frac{20\sin 33^\circ}{\sqrt{151}}$

$B\hat{A}C=62.42949032^\circ$ or $180^\circ-62.42949032^\circ$

$B\hat{A}C=62.4^\circ$ or 118° A2

[4]

(b) $\frac{AB}{\sin A\hat{C}B}=\frac{AC}{\sin A\hat{B}C}$

$\frac{AB}{\sin(180^\circ-33^\circ-62.42949032^\circ)}=\frac{\sqrt{151}}{\sin 33^\circ}$ or

$\frac{AB}{\sin(180^\circ-33^\circ-117.5705097^\circ)}=\frac{\sqrt{151}}{\sin 33^\circ}$ (A1) for substitution

$AB=22.46088256$ or 11.08594015

$AB=22.5$ or 11.1 A2

[3]

(c) 11.4 A1

[1]

3. (a) $\frac{\sin A\hat{B}C}{AC} = \frac{\sin B\hat{A}C}{BC}$ (M1) for sine rule

$\frac{\sin A\hat{B}C}{\sqrt{35}} = \frac{\sin 18^\circ}{2.7}$ (A1) for substitution

$\sin A\hat{B}C = \frac{\sqrt{35}\sin 18^\circ}{2.7}$

$A\hat{B}C = 42.61741708^\circ$ or $180^\circ - 42.61741708^\circ$

$A\hat{B}C = 42.6^\circ$ or 137° A2

[4]

(b) The possible areas

$= \frac{1}{2}(AC)(BC)\sin A\hat{C}B$ (M1) for area formula

$= \frac{1}{2}(\sqrt{35})(2.7)\sin(180^\circ - 18^\circ - 42.61741708^\circ)$ or

$\frac{1}{2}(\sqrt{35})(2.7)\sin(180^\circ - 18^\circ - 137.3825829^\circ)$ (A1) for substitution

$= 6.959321575$ or 3.326920343 (*Rejected*)

Thus, the required area is 6.96. A1

[3]

4. (a) $\dfrac{\sin B\hat{A}C}{BC} = \dfrac{\sin A\hat{B}C}{AC}$ (M1) for sine rule

$\dfrac{\sin B\hat{A}C}{37} = \dfrac{\sin 41^\circ}{27}$ (A1) for substitution

$\sin B\hat{A}C = \dfrac{37 \sin 41^\circ}{27}$

$B\hat{A}C = 64.03266942^\circ$ or $180^\circ - 64.03266942^\circ$

$B\hat{A}C = 64.0^\circ$ or 116° A2

[4]

(b) (i) $AB < 20$ A1

(ii) $\dfrac{AB}{\sin A\hat{C}B} = \dfrac{AC}{\sin A\hat{B}C}$

$\dfrac{AB}{\sin(180^\circ - 41^\circ - 64.03266942^\circ)} = \dfrac{27}{\sin 41^\circ}$ or

$\dfrac{AB}{\sin(180^\circ - 41^\circ - 115.9673306^\circ)} = \dfrac{27}{\sin 41^\circ}$ (A1) for substitution

$AB = 39.74643648$ (*Rejected*) or

$AB = 16.10207244$ A1

The required perimeter

$= 16.10207244 + 27 + 37$

$= 80.10207244$

$= 80.1$ A1

[4]

Chapter 6 Solution

Exercise 19

1. (a) $\frac{1}{3-4i}=\frac{3}{25}+\frac{4}{25}i$ A2

[2]

(b) $z^2=\left(\frac{3}{25}+\frac{4}{25}i\right)^2$

$z^2=-\frac{7}{625}+\frac{24}{625}i$ A2

[2]

(c) $-\frac{7}{625}$ A1

[1]

2. (a) $\frac{z}{1-z}=-1-0.5i$

$z=(-1-0.5i)(1-z)$ (M1) for valid approach

$z=-1+z-0.5i+0.5iz$

$1+0.5i=0.5iz$

$z=\frac{1+0.5i}{0.5i}$ (A1) for correct approach

$z=1-2i$ A1

[3]

(b) -2 A1

[1]

3. (a) $2z-1-i=5+7i$

$2z=6+8i$ (M1) for valid approach

$z=3+4i$ A1

[2]

(b) $z^4=-527-336i$ A2

[2]

(c) -527 A1

[1]

4. (a) $\dfrac{z}{5-12\text{i}} = \dfrac{24-7\text{i}}{\text{i}}$

$z = \dfrac{(24-7\text{i})(5-12\text{i})}{\text{i}}$ — (M1) for valid approach

$z = -323-36\text{i}$ — A1

[2]

(b) $(\text{i}^3 z)^2 = -103033-23256\text{i}$ — A2

[2]

(c) -23256 — A1

[1]

Exercise 20

1. (a) $z = \frac{2-\text{i}}{2+\text{i}}$

$z = \frac{3}{5} - \frac{4}{5}\text{i}$ A2

[2]

(b) The modulus of z

$= \sqrt{\left(\frac{3}{5}\right)^2 + \left(-\frac{4}{5}\right)^2}$ M1

$= \sqrt{1}$

$= 1$ A1

[2]

(c) The argument of z

$= \tan^{-1}\left(\frac{-\frac{4}{5}}{\frac{3}{5}}\right)$ M1

$= \tan^{-1}\left(-\frac{4}{3}\right)$

$= -0.927295218 \text{ rad}$

$= -0.927 \text{ rad}$ A1

[2]

2. (a) $z^2 = \left(\frac{10}{13} - \frac{24}{13}\mathrm{i}\right)^2$

$z^2 = -\frac{476}{169} - \frac{480}{169}\mathrm{i}$ A2

[2]

(b) The modulus of z^2

$= \sqrt{\left(-\frac{476}{169}\right)^2 + \left(-\frac{480}{169}\right)^2}$ M1

$= \sqrt{16}$

$= 4$ A1

[2]

(c) The argument of z^2

$= -\pi + \tan^{-1}\left(\frac{-\frac{480}{169}}{-\frac{476}{169}}\right)$ M1

$= -\pi + \tan^{-1}\left(\frac{120}{119}\right)$

$= -2.352010414 \text{ rad}$

$= -2.35 \text{ rad}$ A1

[2]

3. (a) (i) $z^3 = \left(\frac{8}{5} - \frac{6}{5}\mathrm{i}\right)^3$

$z^3 = -\frac{352}{125} - \frac{936}{125}\mathrm{i}$ A2

(ii) $(z^3)^* + \frac{352}{125} = \frac{936}{125}\mathrm{i}$ A1

[3]

(b) The modulus of $(z^3)^* + \frac{352}{125}$

$= \sqrt{0^2 + \left(\frac{936}{125}\right)^2}$ M1

$= \frac{936}{125}$ A1

[2]

(c) $\frac{\pi}{2}$ rad A1

[1]

4. (a) $|z_1| = |-\mathrm{i}z_2|$

$|z_1| = |-\mathrm{i}||z_2|$ M1

$|z_1| = (1)(2)$

$|z_1| = 2$ A1

[2]

(b) $z_2{}^2 = -2 - 2\sqrt{a}\mathrm{i}$

$|z_2{}^2| = |-2 - 2\sqrt{a}\mathrm{i}|$ M1

$|z_2|^2 = \sqrt{(-2)^2 + (-2\sqrt{a})^2}$ M1

$2^2 = \sqrt{4 + 4a}$

$16 = 4 + 4a$

$12 = 4a$

$a = 3$ A1

[3]

(c) $z_2{}^2 = -2 - 2\sqrt{3}\mathrm{i}$

$\arg(z_2{}^2) = -\pi + \tan^{-1}\left(\dfrac{-2\sqrt{3}}{-2}\right)$ M1

$\arg(z_2{}^2) = -\pi + \tan^{-1}(\sqrt{3})$

$\arg(z_2{}^2) = -\dfrac{2\pi}{3}$ rad A1

[2]

Exercise 21

1. (a) (i) 5 A1

(ii) -12 A1

[2]

(b) The length of AB

$=\sqrt{5^2+(-12)^2}$ M1

$=13$ A1

[2]

(c) The area of ABCD

$=(\text{AB})(\text{AD})$ M1

$=2\text{AB}^2$

$=2(13)^2$

$=338$ A1

[2]

2. (a) (i) $z_B=-18+10\text{i}+20$ M1

$z_B=2+10\text{i}$ A1

(ii) $z_C=-18+10\text{i}+(10-(20\sin 60^\circ)\text{i})$ M1

$z_C=-8+(10-10\sqrt{3})\text{i}$ A1

[4]

(b) The area of ABC

$=\dfrac{1}{2}(20)(20)\sin 60^\circ$ M1

$=173.2050808$

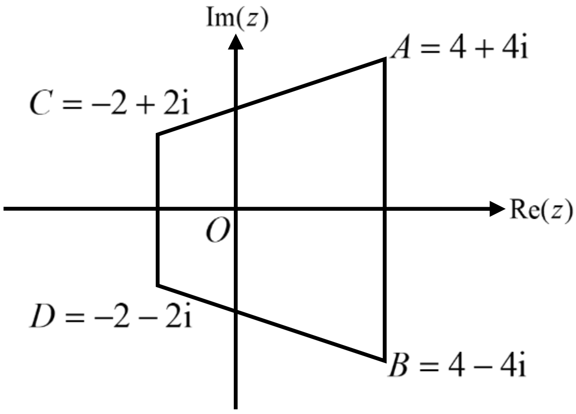
$=173$ A1

[2]

3. (a) For any two correct points A1

For all correct points A1

For sketching a trapezium A1



[3]

(b) $\arg(\omega) = \tan^{-1}\left(\frac{2}{-2}\right)$ M1

$\arg(\omega) = \frac{3\pi}{4}$ rad A1

[2]

(c) The area of the quadrilateral ABDC

$= \frac{(4+8)(6)}{2}$ M1

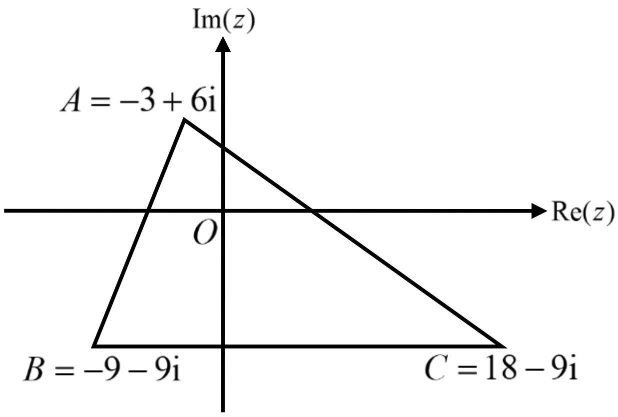
$= 36$ A1

[2]

4. (a) For any two correct points A1

For all correct points A1

For sketching a triangle A1



[3]

(b) $\arg(z-6-15\text{i})=\tan^{-1}\left(\dfrac{-9}{-9}\right)$ M1

$\arg(\omega)=\dfrac{5\pi}{4}$ rad A1

[2]

(c) The area of the triangle ABC

$=\dfrac{(27)(15)}{2}$ M1

$=202.5$ A1

[2]

Exercise 22

1. (a) (i) $\frac{z_1}{z_2} = 12\text{cis}\frac{7\pi}{6} \div \left(4\text{cis}\frac{\pi}{2}\right)$

$\frac{z_1}{z_2} = \frac{12}{4}\text{cis}\left(\frac{7\pi}{6} - \frac{\pi}{2}\right)$ (M1) for valid approach

$\frac{z_1}{z_2} = 3\text{cis}\frac{2\pi}{3}$ A1

(ii) $\frac{z_1}{z_2} = 3e^{\frac{2\pi}{3}\text{i}}$ A1

[3]

(b) The real part of $\frac{z_1}{z_2}$

$= 3\cos\frac{2\pi}{3}$ (M1) for valid approach

$= -\frac{3}{2}$ A1

[2]

2. (a) (i) $z_1 z_2 = \left(18\sqrt{3}\text{cis}\left(-\frac{\pi}{6}\right)\right)\left(\frac{1}{9}\text{cis}\frac{\pi}{3}\right)$

$z_1 z_2 = (18\sqrt{3})\left(\frac{1}{9}\right)\text{cis}\left(-\frac{\pi}{6} + \frac{\pi}{3}\right)$ (M1) for valid approach

$z_1 z_2 = 2\sqrt{3}\text{cis}\frac{\pi}{6}$ A1

(ii) $z_1 z_2 = 2\sqrt{3}e^{\frac{\pi}{6}\text{i}}$ A1

[3]

(b) The real part of $z_1 z_2$

$= 2\sqrt{3}\cos\frac{\pi}{6}$ (M1) for valid approach

$= 3$ A1

[2]

3. (a) (i) $z_1^{\ 2} = \left(2\text{cis}\frac{\pi}{12}\right)^2$

$z_1^{\ 2} = 2^2\text{cis}2\left(\frac{\pi}{12}\right)$ (M1) for valid approach

$z_1^{\ 2} = 4\text{cis}\frac{\pi}{6}$ A1

(ii) The imaginary part of $z_1^{\ 2}$

$= 4\sin\frac{\pi}{6}$ (M1) for valid approach

$= 2$ A1

[4]

(b) (i) $z_1^{\ 2}z_2 = \left(4\text{cis}\frac{\pi}{6}\right)\left(3\text{cis}\frac{\pi}{4}\right)$

$z_1^{\ 2}z_2 = (4)(3)\text{cis}\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$ (M1) for valid approach

$z_1^{\ 2}z_2 = 12\text{cis}\frac{5\pi}{12}$ A1

(ii) $z_1^{\ 2}z_2 = 12e^{\frac{5\pi}{12}\text{i}}$ A1

[3]

4. (a) (i) $z_1^{\ 4} = \left(\frac{1}{3}\text{cis}\frac{\pi}{6}\right)^4$

$z_1^{\ 4} = \left(\frac{1}{3}\right)^4 \text{cis}4\left(\frac{\pi}{6}\right)$ (M1) for valid approach

$z_1^{\ 4} = \frac{1}{81}\text{cis}\frac{2\pi}{3}$ A1

(ii) The real part of $z_1^{\ 4}$

$= \frac{1}{81}\cos\frac{2\pi}{3}$ (M1) for valid approach

$= -\frac{1}{162}$ A1

[4]

(b) (i) $\frac{z_2}{z_1^{\ 4}} = \left(\frac{1}{9}\text{cis}\frac{11\pi}{12}\right) \div \left(\frac{1}{81}\text{cis}\frac{2\pi}{3}\right)$

$\frac{z_2}{z_1^{\ 4}} = \left(\frac{1}{9} \div \frac{1}{81}\right)\text{cis}\left(\frac{11\pi}{12} - \frac{2\pi}{3}\right)$ (M1) for valid approach

$\frac{z_2}{z_1^{\ 4}} = 9\text{cis}\frac{\pi}{4}$ A1

(ii) $\frac{z_2}{z_1^{\ 4}} = 9e^{\frac{\pi}{4}\text{i}}$ A1

[3]

Exercise 23

1. (a) The range of $f(x)$ is $\{y : y \le -25\}$, means the graph of $f(x)$ does not have any x-intercept. R1

[1]

(b) $-x^2 + 4x - 29 = 0$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-29)}}{2(-1)}$$ (A1) for substitution

$$x = \frac{-4 \pm \sqrt{-100}}{-2}$$ (A1) for simplification

$$x = \frac{-4 \pm \sqrt{100}i}{-2}$$

$x = 2 \pm 5i$ A1

[3]

(c) 10 A1

[1]

2. (a) The graph of $f(x)$ opens upward and the vertex is above the x-axis, means the graph of $f(x)$ does not have any x-intercept. R1

[1]

(b) $(x+5)^2 + 64 = 0$

$(x+5)^2 = -64$ (M1) for valid approach

$x + 5 = \pm\sqrt{-64}$ (A1) for correct approach

$x + 5 = \pm\sqrt{64}i$

$x = -5 \pm 8i$ A1

[3]

(c) 16 A1

[1]

3. (a) (i) 8 A1

(ii) 185 A1

[2]

(b) $f(x) = a(x-(4+13\text{i}))(x-(4-13\text{i}))$ (M1) for valid approach

$$f(x) = a\begin{pmatrix} x^2 - ((4+13\text{i})+(4-13\text{i}))x \\ +(4+13\text{i})(4-13\text{i}) \end{pmatrix}$$ (A1) for correct approach

$f(x) = a(x^2 - 8x + 185)$ A1

[3]

(c) $169 = a(4^2 - 8(4) + 185)$ (M1) for setting equation

$169 = 169a$

$a = 1$ A1

[2]

4. (a) (i) -1 A1

(ii) $\frac{17}{4}$ A1

[2]

(b) $f(x) = a\left(x - \left(-\frac{1}{2} + 4\text{i}\right)\right)\left(x - \left(-\frac{1}{2} - 4\text{i}\right)\right)$ (M1) for valid approach

$$f(x) = a\begin{pmatrix} x^2 - \left(\left(-\frac{1}{2}+4\text{i}\right) + \left(-\frac{1}{2}-4\text{i}\right)\right)x \\ +\left(-\frac{1}{2}+4\text{i}\right)\left(-\frac{1}{2}-4\text{i}\right) \end{pmatrix}$$ (A1) for correct approach

$f(x) = a\left(x^2 - (-1)x + \frac{17}{4}\right)$

$f(x) = a\left(x^2 + x + \frac{17}{4}\right)$ A1

[3]

(c) $-17 = a\left(0^2 + 0 + \frac{17}{4}\right)$ (M1) for setting equation

$-17 = \frac{17}{4}a$

$a = -4$ A1

[2]

Exercise 24

1. (a) (i) 2 A1

(ii) 3 A1

[2]

(b) The period of S_2

$= \frac{2\pi}{6}$ (M1) for valid approach

$= \frac{\pi}{3}$ s A1

[2]

(c) $S_1 + S_2 = 2\sin(6t - 0.1) + 3\sin(6t + 0.25)$

$S_1 + S_2 = \text{Im}(2e^{(6t-0.1)i}) + \text{Im}(3e^{(6t+0.25)i})$ (M1) for valid approach

$S_1 + S_2 = \text{Im}(2e^{(6t-0.1)i} + 3e^{(6t+0.25)i})$ (A1) for correct approach

$S_1 + S_2 = \text{Im}(e^{6ti}(2e^{-0.1i} + 3e^{0.25i}))$

$\therefore z + w = 2e^{-0.1i} + 3e^{0.25i}$ A1

[3]

(d) (i) $z = 2e^{-0.1i}$

$z = 2(\cos(-0.1) + i\sin(-0.1))$ A1

(ii) $w = 3e^{0.25i}$

$w = 3(\cos 0.25 + i\sin 0.25)$ A1

[2]

(e) (i) $z + w = 2(\cos(-0.1) + i\sin(-0.1))$

$+3(\cos 0.25 + i\sin 0.25)$

$z + w = (2\cos(-0.1) + 3\cos 0.25)$

$+i(2\sin(-0.1) + 3\sin 0.25)$ (M1) for valid approach

$z + w = 4.896745596 + 0.5425450445i$ (A1) for correct values

$L = \sqrt{4.896745596^2 + 0.5425450445^2}$ M1

$L = 4.926710115$

$L = 4.93$ A1

(ii) $\alpha = \tan^{-1}\frac{0.5425450445}{4.896745596}$ M1

$\alpha = 0.1103469951$

$\alpha = 0.110$ A1

[6]

(f) $S_1 + S_2 = \text{Im}(e^{6ti}(z+w))$

$S_1 + S_2 = \text{Im}(e^{6ti} \cdot 4.926710115e^{0.1103469951i})$ (M1) for substitution

$S_1 + S_2 = \text{Im}(4.926710115e^{6ti+0.1103469951i})$ (A1) for correct approach

$S_1 + S_2 = 4.926710115\sin(6t + 0.1103469951)$

$S_1 + S_2 = 4.93\sin(6t + 0.110)$ A1

[3]

(g) -4.93 mm A1

[1]

2. (a) (i) 5 A1

(ii) 7 A1

[2]

(b) The period of W_2

$$= \frac{2\pi}{\pi}$$ (M1) for valid approach

$= 2$ s A1

[2]

(c) $W_1 + W_2 = 5\cos(\pi t - 0.9) + 7\cos(\pi t - 1.3)$

$W_1 + W_2 = \text{Re}(5e^{(\pi t - 0.9)\text{i}}) + \text{Re}(7e^{(\pi t - 1.3)\text{i}})$ (M1) for valid approach

$W_1 + W_2 = \text{Re}(5e^{(\pi t - 0.9)\text{i}} + 7e^{(\pi t - 1.3)\text{i}})$ (A1) for correct approach

$W_1 + W_2 = \text{Re}(e^{\pi t\text{i}}(5e^{-0.9\text{i}} + 7e^{-1.3\text{i}}))$

$\therefore z + w = 5e^{-0.9\text{i}} + 7e^{-1.3\text{i}}$ A1

[3]

(d) (i) $z = 5e^{-0.9\text{i}}$

$z = 5(\cos(-0.9) + \text{i}\sin(-0.9))$ A1

(ii) $w = 7e^{-1.3\text{i}}$

$w = 7(\cos(-1.3) + \text{i}\sin(-1.3))$ A1

[2]

(e) (i) $z + w = 5(\cos(-0.9) + \text{i}\sin(-0.9))$

$+7(\cos(-1.3) + \text{i}\sin(-1.3))$

$z + w = (5\cos(-0.9) + 7\cos(-1.3))$

$+\text{i}(5\sin(-0.9) + 7\sin(-1.3))$ (M1) for valid approach

$z + w = 4.980541642 - 10.66154185\text{i}$ (A1) for correct values

$L = \sqrt{4.980541642^2 + (-10.66154185)^2}$ M1

$L = 11.76750907$

$L = 11.8$ A1

(ii) $\alpha = \tan^{-1}\frac{-10.66154185}{4.980541642}$ M1

$\alpha = -1.13377216$

$\alpha = -1.13$ A1

[6]

(f) $W_1 + W_2 = \text{Re}(e^{\pi t \text{i}}(z+w))$

$W_1 + W_2 = \text{Re}(e^{\pi t \text{i}} \cdot 11.76750907 e^{-1.13377216\text{i}})$ (M1) for substitution

$W_1 + W_2 = \text{Re}(11.76750907 e^{\pi t \text{i} - 1.13377216\text{i}})$ (A1) for correct approach

$W_1 + W_2 = 11.76750907 \cos(\pi t - 1.13377216)$

$W_1 + W_2 = 11.8 \cos(\pi t - 1.13)$ A1

[3]

(g) $W = 0$

$11.76750907 \cos(\pi t - 1.13377216) = 0$ (M1) for setting equation

By considering the graph of

$y = 11.76750907 \cos(\pi t - 1.13377216)$,

$t = 1.8608909$.

$\therefore t = 1.86$ A1

[2]

3. (a) (i) 10 A1

(ii) $\frac{\pi}{5}$ s A1

[2]

(b) $S_2 = S - S_1$

$S_2 = 10\cos(10t + 0.15) - 8\cos(10t + 0.05)$

$S_2 = \text{Re}(10e^{(10t+0.15)\text{i}}) - \text{Re}(8e^{(10t+0.05)\text{i}})$ (M1) for valid approach

$S_2 = \text{Re}(10e^{(10t+0.15)\text{i}} - 8e^{(10t+0.05)\text{i}})$ (A1) for correct approach

$S_2 = \text{Re}(e^{10t\text{i}}(10e^{0.15\text{i}} - 8e^{0.05\text{i}}))$

$\therefore z - w = 10e^{0.15\text{i}} - 8e^{0.05\text{i}}$ A1

[3]

(c) (i) $z = 10e^{0.15\text{i}}$

$z = 10(\cos 0.15 + \text{i}\sin 0.15)$ A1

(ii) $w = 8e^{0.05\text{i}}$

$w = 8(\cos 0.05 + \text{i}\sin 0.05)$ A1

[2]

(d) (i) $z - w = 10(\cos 0.15 + \text{i}\sin 0.15)$

$-8(\cos 0.05 + \text{i}\sin 0.05)$

$z - w = (10\cos 0.15 - 8\cos 0.05)$

$+\text{i}(10\sin 0.15 - 8\sin 0.05)$ (M1) for valid approach

$z - w = 1.897708696 + 1.094547971\text{i}$ (A1) for correct values

$L = \sqrt{1.897708696^2 + 1.094547971^2}$ M1

$L = 2.19073813$

$L = 2.19$ A1

(ii) $\alpha = \tan^{-1}\frac{1.094547971}{1.897708696}$ M1

$\alpha = 0.523166045$

$\alpha = 0.523$ A1

[6]

(e) $S_2 = \text{Re}(e^{10t\text{i}}(z - w))$

$S_2 = \text{Re}(e^{10t\text{i}} \cdot 2.19073813e^{0.523166045\text{i}})$ (M1) for substitution

$S_2 = \text{Re}(2.19073813e^{10t\text{i}+0.523166045\text{i}})$ (A1) for correct approach

$S_2 = 2.19073813\cos(10t + 0.523166045)$

$S_2 = 2.19\cos(10t + 0.523)$ A1

[3]

(f) $S_2 = 1.5$

$2.19073813\cos(10t + 0.523166045) = 1.5$ (M1) for setting equation

$2.19073813\cos(10t + 0.523166045) - 1.5 = 0$

By considering the graph of

$y = 2.19073813\cos(10t + 0.523166045) - 1.5$,

$t = 9.9191196$.

$\therefore t = 9.92$ A1

[2]

4. (a) (i) 7 A1

(ii) 1 s A1

[2]

(b) $V_2 = V - V_1$

$V_2 = 6.3\sin(2\pi t - 0.5) - 7\sin(2\pi t - 0.95)$

$V_2 = \text{Im}(6.3e^{(2\pi t - 0.5)\text{i}}) - \text{Im}(7e^{(2\pi t - 0.95)\text{i}})$ (M1) for valid approach

$V_2 = \text{Im}(6.3e^{(2\pi t - 0.5)\text{i}} - 7e^{(2\pi t - 0.95)\text{i}})$ (A1) for correct approach

$V_2 = \text{Im}(e^{2\pi t\text{i}}(6.3e^{-0.5\text{i}} - 7e^{-0.95\text{i}}))$

$\therefore z - w = 6.3e^{-0.5\text{i}} - 7e^{-0.95\text{i}}$ A1

[3]

(c) (i) $z = 6.3e^{-0.5\text{i}}$

$z = 6.3(\cos(-0.5) + \text{i}\sin(-0.5))$ A1

(ii) $w = 7e^{-0.95\text{i}}$

$w = 7(\cos(-0.95) + \text{i}\sin(-0.95))$ A1

[2]

(d) (i) $z - w = 6.3(\cos(-0.5) + \text{i}\sin(-0.5))$

$-7(\cos(-0.95) + \text{i}\sin(-0.95))$

$z - w = (6.3\cos(-0.5) - 7\cos(-0.95))$

$+\text{i}(6.3\sin(-0.5) - 7\sin(-0.95))$ (M1) for valid approach

$z - w = 1.456988514 + 2.67352764\text{i}$ (A1) for correct values

$L = \sqrt{1.456988514^2 + 2.67352764^2}$ M1

$L = 3.044760347$

$L = 3.04$ A1

(ii) $\alpha = \tan^{-1}\dfrac{2.67352764}{1.456988514}$ M1

$\alpha = 1.071824229$

$\alpha = 1.07$ A1

[6]

(e) $V_2 = \text{Im}(e^{2\pi t\text{i}}(z - w))$

$V_2 = \text{Im}(e^{2\pi t\text{i}} \cdot 3.044760347e^{1.071824229\text{i}})$ (M1) for substitution

$V_2 = \text{Im}(3.044760347e^{2\pi t\text{i} + 1.071824229\text{i}})$ (A1) for correct approach

$V_2 = 3.044760347\sin(2\pi t + 1.071824229)$

$V_2 = 3.04\sin(2\pi t + 1.07)$ A1

[3]

(f) $V_2 > 2$

$3.044760347\sin(2\pi t + 1.071824229) > 2$ (M1) for setting inequality

$3.044760347\sin(2\pi t + 1.071824229) - 2 > 0$

By considering the graph of

$y = 3.044760347\sin(2\pi t + 1.071824229) - 2$,

$0.9434731 < t < 1.2153547$. (A1) for correct values

$\therefore 0.943 < t < 1.22$ A1

[3]

Chapter 7 Solution

Exercise 25

1. (a) 15 — A1

[1]

(b) $\det \mathbf{B} = (2x^2)(x) - (4)(x)$ — (A1) for substitution

$\det \mathbf{B} = 2x^3 - 4x$ — A1

[2]

(c) $(11 - \det \mathbf{A})x = \det \mathbf{B}$

$\therefore (11 - 15)x = 2x^3 - 4x$ — (M1) for setting equation

$-4x = 2x^3 - 4x$

$2x^3 = 0$

$x = 0$ — A1

[2]

2. (a) 3 — A1

[1]

(b) $\det \mathbf{B} = (1 + x)(1 - x) - (x)(-2x)$ — (A1) for substitution

$\det \mathbf{B} = 1 - x^2 + 2x^2$

$\det \mathbf{B} = 1 + x^2$ — A1

[2]

(c) $\det \mathbf{A} + \det \mathbf{B} - 5x = 0$

$\therefore 3 + 1 + x^2 - 5x = 0$ — (M1) for setting equation

$x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0$

$x = 1$ or $x = 4$ — A2

[3]

3.	(a)	$\det \mathbf{A} = (4)(e^{2x}) - (1)(3e^{x})$	(A1) for substitution
		$\det \mathbf{A} = 4e^{2x} - 3e^{x}$	A1
			[2]
	(b)	$\det \mathbf{A} - 1 = 0$	
		$\therefore 4e^{2x} - 3e^{x} - 1 = 0$	(M1) for setting equation
		$(4e^{x} + 1)(e^{x} - 1) = 0$	(A1) for factorization
		$e^{x} = -\frac{1}{4}$ (*Rejected*) or $e^{x} = 1$	
		$\therefore x = 0$	A1
			[3]
4.	(a)	$\det \mathbf{A} = (\ln x)(\ln x) - (3)(-2)$	(A1) for substitution
		$\det \mathbf{A} = (\ln x)^2 + 6$	A1
			[2]
	(b)	$\det \mathbf{A} = 5\ln x$	
		$\therefore (\ln x)^2 + 6 = 5\ln x$	(M1) for setting equation
		$(\ln x)^2 - 5\ln x + 6 = 0$	
		$(\ln x - 2)(\ln x - 3) = 0$	(A1) for factorization
		$\ln x = 2$ or $\ln x = 3$	
		$\therefore x = e^2$ or $x = e^3$	A2
			[4]

Exercise 26

1. (a) $\mathbf{A}^{-1} = \begin{pmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ -1 & -\frac{7}{3} & -\frac{11}{3} \\ -2 & -5 & -7 \end{pmatrix}$ A2

[2]

(b) $\mathbf{AB} + \begin{pmatrix} 1 & -2 & -2 \\ 0 & 1 & 3 \\ -1 & 0 & -3 \end{pmatrix} = 2\mathbf{I}$

$\mathbf{AB} = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & -3 \\ 1 & 0 & 5 \end{pmatrix}$

$\mathbf{B} = \mathbf{A}^{-1}\begin{pmatrix} 1 & 2 & 2 \\ 0 & 1 & -3 \\ 1 & 0 & 5 \end{pmatrix}$ (M1) for valid approach

$\mathbf{B} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & \frac{7}{3} \\ -\frac{14}{3} & -\frac{13}{3} & -\frac{40}{3} \\ -9 & -9 & -24 \end{pmatrix}$ A2

[3]

2. (a) $\mathbf{A}^{-1} = \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} & -\frac{1}{5} \end{pmatrix}$ A2

[2]

(b) $\mathbf{AB} = \begin{pmatrix} -8 & 5 & 3 \\ 2 & 6 & 7 \\ 5 & -4 & -4 \end{pmatrix} - 5\mathbf{I}$

$\mathbf{AB} = \begin{pmatrix} -13 & 5 & 3 \\ 2 & 1 & 7 \\ 5 & -4 & -9 \end{pmatrix}$

$\mathbf{B} = \mathbf{A}^{-1}\begin{pmatrix} -13 & 5 & 3 \\ 2 & 1 & 7 \\ 5 & -4 & -9 \end{pmatrix}$ (M1) for valid approach

$\mathbf{B} = \begin{pmatrix} -\frac{33}{5} & \frac{13}{5} & \frac{8}{5} \\ \frac{34}{5} & -\frac{14}{5} & -\frac{9}{5} \\ \frac{42}{5} & -\frac{7}{5} & \frac{28}{5} \end{pmatrix}$ A2

[3]

3. (a) $\mathbf{A}^{-1} = \begin{pmatrix} -\frac{5}{2} & 1 & \frac{9}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{9}{4} & -1 & -\frac{15}{4} \end{pmatrix}$ A2

[2]

(b) $\mathbf{A}^{-1}\mathbf{CA} = \frac{1}{2}\mathbf{B}$

$\mathbf{C} = \mathbf{A}\left(\frac{1}{2}\mathbf{B}\right)\mathbf{A}^{-1}$ (M1) for valid approach

$\mathbf{C} = \frac{1}{2}\mathbf{ABA}^{-1}$

$\mathbf{C} = \begin{pmatrix} \frac{1}{4} & -\frac{11}{2} & -\frac{7}{4} \\ -\frac{113}{4} & \frac{21}{2} & \frac{177}{4} \\ \frac{15}{4} & -\frac{9}{2} & -\frac{25}{4} \end{pmatrix}$ A2

[3]

4. (a) $\mathbf{A}^{-1} = \begin{pmatrix} \frac{29}{2} & -\frac{53}{2} & 1 \\ \frac{3}{2} & -\frac{5}{2} & 0 \\ -1 & 2 & 0 \end{pmatrix}$ A2

[2]

(b) $\mathbf{ACA}^{-1} = \mathbf{B}^3$

$\mathbf{C} = \mathbf{A}^{-1}\mathbf{B}^3\mathbf{A}$ (M1) for valid approach

$\mathbf{C} = \begin{pmatrix} -956 & 4131 & -6559.5 \\ -92 & 403 & -624.5 \\ 71 & -303 & 492 \end{pmatrix}$ A2

[3]

Exercise 27

1. (a) $$\mathbf{A}^{-1} = \begin{pmatrix} \frac{6}{13} & \frac{8}{39} & -\frac{29}{39} \\ -\frac{5}{13} & \frac{2}{39} & \frac{22}{39} \\ \frac{1}{13} & -\frac{1}{13} & \frac{2}{13} \end{pmatrix}$$ A2

[2]

(b) $\mathbf{AX} = \mathbf{B}$

$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ (M1) for valid approach

$$\mathbf{X} = \begin{pmatrix} -\frac{170}{39} \\ \frac{133}{39} \\ \frac{5}{13} \end{pmatrix}$$ A2

[3]

2. (a) $\mathbf{A}^{-1} = \begin{pmatrix} -\frac{14}{37} & \frac{5}{37} & \frac{10}{37} \\ -\frac{45}{37} & \frac{24}{37} & \frac{11}{37} \\ -\frac{13}{37} & \frac{2}{37} & \frac{4}{37} \end{pmatrix}$ A2

[2]

(b) $\begin{cases} 2x - 5z = 740 \\ x + 2y - 8z = 592 \\ 6x - y - 3z = -444 \end{cases}$

$$\begin{pmatrix} 2 & 0 & -5 \\ 1 & 2 & -8 \\ 6 & -1 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 740 \\ 592 \\ -444 \end{pmatrix}$$ (M1) for valid approach

$$\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 740 \\ 592 \\ -444 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} 740 \\ 592 \\ -444 \end{pmatrix}$$ (M1) for valid approach

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -320 \\ -648 \\ -276 \end{pmatrix}$$

$\therefore x = -320,\ y = -648,\ z = -276$ A2

[4]

3. (a) $\mathbf{A} = (\mathbf{A}^{-1})^{-1}$ (M1) for valid approach

$$\mathbf{A} = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$\therefore a = 2$ A1

[2]

(b) $\mathbf{AX} = \mathbf{B}$

$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ (M1) for valid approach

$$\mathbf{X} = \begin{pmatrix} -6 \\ 15 \\ -12 \end{pmatrix}$$ A2

[3]

4. (a) $\mathbf{A} = (\mathbf{A}^{-1})^{-1}$ (M1) for valid approach

$$\mathbf{A} = \begin{pmatrix} 8 & 16 & 16 \\ 8 & -8 & -24 \\ 8 & -16 & -16 \end{pmatrix}$$

$\therefore p = 8,\ q = -24$ A2

[3]

(b) $\mathbf{AX} = \mathbf{B}$

$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ (M1) for valid approach

$$\mathbf{X} = \begin{pmatrix} \frac{1}{16} \\ -\frac{15}{64} \\ \frac{9}{64} \end{pmatrix}$$ A2

[3]

Exercise 28

1. (a) $\mathbf{ST}=\begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 3 & 0\end{pmatrix}$

$\mathbf{ST}=\begin{pmatrix}6 & 0\\ 0 & 1\end{pmatrix}$ A2

[2]

(b) Horizontal stretch with scale factor 6. A1

[1]

(c) $\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}6 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}3\\ -5\end{pmatrix}$ (M1) for valid approach

$\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}18\\ -5\end{pmatrix}$

Thus, the coordinates of P are $(18,-5)$. A1

[2]

2. (a) $\mathbf{ST}=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$\mathbf{ST}=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$ A2

[2]

(b) Reflection about the line $x=0$. A1

[1]

(c) $\begin{pmatrix}2\\ 1\end{pmatrix}=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$ (M1) for valid approach

$\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}^{-1}\begin{pmatrix}2\\ 1\end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-2\\ 1\end{pmatrix}$

Thus, the coordinates of P are $(-2,1)$. A1

[3]

3. (a) Rotation clockwise of $\frac{\pi}{6}$ radians about the origin. A1

[1]

(b) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ (M1) for valid approach

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5.464101615 \\ 1.464101615 \end{pmatrix}$

Thus, the coordinates of P are $(5.46, 1.46)$. A1

[2]

(c) 12 A2

[2]

4. (a) Rotation anticlockwise of $\frac{2\pi}{3}$ radians about the origin. A1

[1]

(b) $\begin{pmatrix} -2 \\ 2\sqrt{3} \end{pmatrix} = \mathbf{T} \begin{pmatrix} x \\ y \end{pmatrix}$ (M1) for valid approach

$\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{T}^{-1} \begin{pmatrix} -2 \\ 2\sqrt{3} \end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

Thus, the coordinates of P are $(4, 0)$. A1

[3]

(c) 3 A2

[2]

Exercise 29

1. (a) The characteristic polynomial of $\mathbf{A}$

$= \det(\mathbf{A} - \lambda\mathbf{I})$

$= \begin{vmatrix} -2-\lambda & -3 \\ 1 & 2-\lambda \end{vmatrix}$ (M1) for valid approach

$= (-2-\lambda)(2-\lambda) - (-3)(1)$

$= -4 + 2\lambda - 2\lambda + \lambda^2 + 3$

$= \lambda^2 - 1$ A1

[2]

(b) $\lambda_1 = -1,\ \lambda_2 = 1$ A2

[2]

(c) $\mathbf{v}_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix},\ \mathbf{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ A2

[2]

(d) $\det(\mathbf{A}) = \dfrac{\alpha}{\lambda_1 \lambda_2}$

$\therefore -1 = \dfrac{\alpha}{(-1)(1)}$ (M1) for setting equation

$\alpha = 1$ A1

[2]

(e) (i) $\begin{pmatrix} -3 & -1 \\ 1 & 1 \end{pmatrix}$ A1

(ii) $\begin{pmatrix} (-1)^n & 0 \\ 0 & 1 \end{pmatrix}$ A2

[3]

(f) $\mathbf{A}^n = \begin{pmatrix} -3 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} (-1)^n & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -3 & -1 \\ 1 & 1 \end{pmatrix}^{-1}$

$\mathbf{A}^n = \begin{pmatrix} -3(-1)^n & -1 \\ (-1)^n & 1 \end{pmatrix} \begin{pmatrix} -3 & -1 \\ 1 & 1 \end{pmatrix}^{-1}$ A1

$\mathbf{A}^n = \begin{pmatrix} -3(-1)^n & -1 \\ (-1)^n & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$ (A1) for correct approach

$\mathbf{A}^n = \begin{pmatrix} \frac{3}{2}(-1)^n - \frac{1}{2} & \frac{3}{2}(-1)^n - \frac{3}{2} \\ -\frac{1}{2}(-1)^n + \frac{1}{2} & -\frac{1}{2}(-1)^n + \frac{3}{2} \end{pmatrix}$ A1

[3]

2. (a) $\det(\mathbf{A}-\lambda\mathbf{I}) = \begin{vmatrix} 9-\lambda & -4 \\ 2 & 3-\lambda \end{vmatrix}$ (M1) for valid approach

$\det(\mathbf{A}-\lambda\mathbf{I}) = (9-\lambda)(3-\lambda)-(-4)(2)$

$\det(\mathbf{A}-\lambda\mathbf{I}) = 27-9\lambda-3\lambda+\lambda^2+8$

$\det(\mathbf{A}-\lambda\mathbf{I}) = \lambda^2-12\lambda+35$ A1

[2]

(b) $\lambda_1 = 5$, $\lambda_2 = 7$ A2

[2]

(c) $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ A2

[2]

(d) $3\det(\mathbf{A})+\alpha\lambda_1\lambda_2 = 0$

$\therefore 3(35)+\alpha(5)(7) = 0$ (M1) for setting equation

$35\alpha = -105$

$\alpha = -3$ A1

[2]

(e) (i) $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ A1

(ii) $\begin{pmatrix} 5^n & 0 \\ 0 & 7^n \end{pmatrix}$ A2

[3]

(f) $\mathbf{A}^{10} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 5^{10} & 0 \\ 0 & 7^{10} \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}^{-1}$

$\mathbf{A}^{10} = \begin{pmatrix} 9765625 & 564950498 \\ 9765625 & 282475249 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}^{-1}$ A1

$\mathbf{A}^{10} = \begin{pmatrix} 9765625 & 564950498 \\ 9765625 & 282475249 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$ (A1) for correct approach

$\mathbf{A}^{10} = \begin{pmatrix} 555184873 & -545419248 \\ 272709624 & -262943999 \end{pmatrix}$ A1

[3]

3. (a) The characteristic polynomial of $\mathbf{M}$

$$= \det(\mathbf{M} - \lambda \mathbf{I})$$

$$= \begin{vmatrix} -1-\lambda & \frac{1}{16} \\ -35 & 2-\lambda \end{vmatrix}$$ (M1) for valid approach

$$= (-1-\lambda)(2-\lambda) - \left(\frac{1}{16}\right)(-35)$$

$$= -2 + \lambda - 2\lambda + \lambda^2 + \frac{35}{16}$$

$$= \lambda^2 - \lambda + \frac{3}{16}$$ A1

[2]

(b) $\lambda_1 = \frac{1}{4},\ \lambda_2 = \frac{3}{4}$ A2

[2]

(c) $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 20 \end{pmatrix},\ \mathbf{v}_2 = \begin{pmatrix} 1 \\ 28 \end{pmatrix}$ A2

[2]

(d) (i) $\begin{pmatrix} 1 & 1 \\ 20 & 28 \end{pmatrix}$ A1

(ii) $\begin{pmatrix} \left(\frac{1}{4}\right)^n & 0 \\ 0 & \left(\frac{3}{4}\right)^n \end{pmatrix}$ A2

[3]

(e) $\mathbf{M}^n = \begin{pmatrix} 1 & 1 \\ 20 & 28 \end{pmatrix} \begin{pmatrix} \left(\frac{1}{4}\right)^n & 0 \\ 0 & \left(\frac{3}{4}\right)^n \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 20 & 28 \end{pmatrix}^{-1}$

$\mathbf{M}^n = \begin{pmatrix} \left(\frac{1}{4}\right)^n & \left(\frac{3}{4}\right)^n \\ 20\left(\frac{1}{4}\right)^n & 28\left(\frac{3}{4}\right)^n \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 20 & 28 \end{pmatrix}^{-1}$ A1

$\mathbf{M}^n = \begin{pmatrix} \left(\frac{1}{4}\right)^n & \left(\frac{3}{4}\right)^n \\ 20\left(\frac{1}{4}\right)^n & 28\left(\frac{3}{4}\right)^n \end{pmatrix} \begin{pmatrix} \frac{7}{2} & -\frac{1}{8} \\ -\frac{5}{2} & \frac{1}{8} \end{pmatrix}$ (A1) for correct approach

$\mathbf{M}^n = \begin{pmatrix} \frac{7}{2}\left(\frac{1}{4}\right)^n - \frac{5}{2}\left(\frac{3}{4}\right)^n & -\frac{1}{8}\left(\frac{1}{4}\right)^n + \frac{1}{8}\left(\frac{3}{4}\right)^n \\ 70\left(\frac{1}{4}\right)^n - 70\left(\frac{3}{4}\right)^n & -\frac{5}{2}\left(\frac{1}{4}\right)^n + \frac{7}{2}\left(\frac{3}{4}\right)^n \end{pmatrix}$ A1

[3]

(f) $\lim_{n\to\infty} f(n) = 0$ A1

[1]

4. (a) The characteristic polynomial of $\mathbf{M}$

$= \det(\mathbf{M} - \lambda\mathbf{I})$

$= \begin{vmatrix} \frac{3}{2} - \lambda & -\frac{1}{2} \\ 1 & 0 - \lambda \end{vmatrix}$ (M1) for valid approach

$= \left(\frac{3}{2} - \lambda\right)(0 - \lambda) - \left(-\frac{1}{2}\right)(1)$

$= -\frac{3}{2}\lambda + \lambda^2 + \frac{1}{2}$

$= \lambda^2 - \frac{3}{2}\lambda + \frac{1}{2}$ A1

[2]

(b) $\lambda_1 = \frac{1}{2},\ \lambda_2 = 1$ A2

[2]

(c) $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix},\ \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ A2

[2]

(d) (i) $\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$ A1

(ii) $\begin{pmatrix} \left(\frac{1}{2}\right)^n & 0 \\ 0 & 1 \end{pmatrix}$ A2

[3]

(e) $\mathbf{M}^n = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \left(\frac{1}{2}\right)^n & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}^{-1}$

$\mathbf{M}^n = \begin{pmatrix} \left(\frac{1}{2}\right)^n & 1 \\ 2\left(\frac{1}{2}\right)^n & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}^{-1}$ A1

$\mathbf{M}^n = \begin{pmatrix} \left(\frac{1}{2}\right)^n & 1 \\ 2\left(\frac{1}{2}\right)^n & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$ (A1) for correct approach

$\mathbf{M}^n = \begin{pmatrix} -\left(\frac{1}{2}\right)^n + 2 & \left(\frac{1}{2}\right)^n - 1 \\ -2\left(\frac{1}{2}\right)^n + 2 & 2\left(\frac{1}{2}\right)^n - 1 \end{pmatrix}$ A1

[3]

(f) $\lim_{n \to \infty} g(n) = -1$ A1

[1]

Exercise 30

1.	(a)	(i)	$3600 = a(50)^2 + b(50) + c$	A1
			$2500a + 50b + c = 3600$	AG
		(ii)	$400a + 20b + c = -900$	A1
			$25a + 5b + c = -1125$	A1
				[3]
	(b)	(i)	$\mathbf{A} = \begin{pmatrix} 2500 & 50 & 1 \\ 400 & 20 & 1 \\ 25 & 5 & 1 \end{pmatrix}$	A1
		(ii)	$\mathbf{B} = \begin{pmatrix} 3600 \\ -900 \\ -1125 \end{pmatrix}$	A1
		(iii)	$\mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{1350} & -\frac{1}{450} & \frac{1}{675} \\ -\frac{1}{54} & \frac{11}{90} & -\frac{14}{135} \\ \frac{2}{27} & -\frac{5}{9} & \frac{40}{27} \end{pmatrix}$	A2
				[4]
	(c)		$a = 3$, $b = -60$ and $c = -900$	
			For any one correct answer	A1
			For all correct answers	A1
				[2]
	(d)	(i)	$3x^2 - 60x - 900 = 0$	
			$3(x+10)(x-30) = 0$	(A1) for factorization
			$x = -10$ or $x = 30$	A2
		(ii)	The y-coordinate of the vertex	
			$= 3\left(\frac{-10+30}{2}\right)^2 - 60\left(\frac{-10+30}{2}\right) - 900$	(M1) for substitution
			$= -1200$	A1
				[5]

2.	(a)	(i)	$-384 = a(0)^3 + b(0)^2 + c(0) + d$	A1
			$d = -384$	AG
		(ii)	$-840 = a(2)^3 + b(2)^2 + c(2) - 384$	A1
			$-456 = 8a + 4b + 2c$	
			$4a + 2b + c = -228$	AG
		(iii)	$36a + 6b + c = -356$	A1
			$100a + 10b + c = -516$	A1

[4]

(b)	(i)	$\mathbf{A} = \begin{pmatrix} 4 & 2 & 1 \\ 36 & 6 & 1 \\ 100 & 10 & 1 \end{pmatrix}$	A1
	(ii)	$\mathbf{B} = \begin{pmatrix} -228 \\ -356 \\ -516 \end{pmatrix}$	A1
	(iii)	$\mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{32} & -\frac{1}{16} & \frac{1}{32} \\ -\frac{1}{2} & \frac{3}{4} & -\frac{1}{4} \\ \frac{15}{8} & -\frac{5}{4} & \frac{3}{8} \end{pmatrix}$	A2

[4]

(c) $a = -1,\ b = -24$ and $c = -176$

For any one correct answer	A1
For all correct answers	A1

[2]

(d)	(i)	$x = -12,\ x = -8,\ x = -4$	A3
	(ii)	$y = -384$	A1

[4]

3. (a) (i) $\mathbf{M}^2 = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$

$\mathbf{M}^2 = \begin{pmatrix} 1 & 0 \\ 10 & 1 \end{pmatrix}$ A2

(ii) $\mathbf{M}^3 = \begin{pmatrix} 1 & 0 \\ 10 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$

$\mathbf{M}^3 = \begin{pmatrix} 1 & 0 \\ 15 & 1 \end{pmatrix}$ A2

(iii) $\mathbf{M}^{50} = \begin{pmatrix} 1 & 0 \\ 250 & 1 \end{pmatrix}$ A1

[5]

(b) (i) $s(2) = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 10 & 1 \end{pmatrix}$

$s(2) = \begin{pmatrix} 2 & 0 \\ 15 & 2 \end{pmatrix}$ A1

(ii) $s(3) = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 10 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 15 & 1 \end{pmatrix}$

$s(3) = \begin{pmatrix} 3 & 0 \\ 30 & 3 \end{pmatrix}$ A1

(iii) $s(50) = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 10 & 1 \end{pmatrix} + \cdots + \begin{pmatrix} 1 & 0 \\ 250 & 1 \end{pmatrix}$

$s(50) = \begin{pmatrix} 50 & 0 \\ 5+10+\cdots+250 & 50 \end{pmatrix}$ (M1) for valid approach

$s(50) = \begin{pmatrix} 50 & 0 \\ \frac{50}{2}(5+250) & 50 \end{pmatrix}$ M1A1

$s(50) = \begin{pmatrix} 50 & 0 \\ 6375 & 50 \end{pmatrix}$ A1

[6]

(c) $p(50) = \mathbf{M} \times \mathbf{M}^2 \times \mathbf{M}^3 \times \cdots \times \mathbf{M}^{50}$

$p(50) = \mathbf{M}^{1+2+\cdots+50}$ (A1) for correct approach

$p(50) = \mathbf{M}^{\frac{50}{2}(1+50)}$ M1A1

$p(50) = \mathbf{M}^{1275}$

$p(50) = \begin{pmatrix} 1 & 0 \\ 6375 & 1 \end{pmatrix}$ A1

[4]

4. (a) (i) $\mathbf{A}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

$\mathbf{A}^2 = \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}$ A2

(ii) $\mathbf{A}^3 = \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

$\mathbf{A}^3 = \begin{pmatrix} 1 & 12 \\ 0 & 1 \end{pmatrix}$ A2

(iii) $\mathbf{A}^{30} = \begin{pmatrix} 1 & 120 \\ 0 & 1 \end{pmatrix}$ A1

[5]

(b) (i) $\mathbf{B} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 3 \\ 0 & 1 \end{pmatrix}$

$\mathbf{B} = \begin{pmatrix} 1 & 7 \\ 0 & 2 \end{pmatrix}$ A1

$\mathbf{B}^2 = \begin{pmatrix} 1 & 7 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 7 \\ 0 & 2 \end{pmatrix}$

$\mathbf{B}^2 = \begin{pmatrix} (1)(1)+(7)(0) & (1)(7)+(7)(2) \\ (0)(1)+(2)(0) & (0)(7)+(2)(2) \end{pmatrix}$ A1

$\mathbf{B}^2 = \begin{pmatrix} 1 & 21 \\ 0 & 4 \end{pmatrix}$ AG

(ii) $\mathbf{B}^3 = \mathbf{B} \times \mathbf{B}^2$

$\mathbf{B}^3 = \begin{pmatrix} 1 & 7 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 21 \\ 0 & 4 \end{pmatrix}$

$\mathbf{B}^3 = \begin{pmatrix} 1 & 49 \\ 0 & 8 \end{pmatrix}$ A2

(iii) $\mathbf{B}^{30}=\begin{pmatrix}1 & 7+14+28+\cdots \\ 0 & 2^{30}\end{pmatrix}$ (M1) for valid approach

$\mathbf{B}^{30}=\begin{pmatrix}1 & \dfrac{7(2^{30}-1)}{2-1} \\ 0 & 1073741824\end{pmatrix}$ M1A1

$\mathbf{B}^{30}=\begin{pmatrix}1 & 7516192761 \\ 0 & 1073741824\end{pmatrix}$ A1

[8]

(c) When $n=30$, $\det(\mathbf{B}^{30})=1073741824$,

$\det(\mathbf{A}^{30})=1$ and $\det\left(\begin{pmatrix}0 & 3 \\ 0 & 1\end{pmatrix}^{30}\right)=0$. R1

Thus, $\det(\mathbf{B}^{n})=\det(\mathbf{A}^{n})+\det\left(\begin{pmatrix}0 & 3 \\ 0 & 1\end{pmatrix}^{n}\right)$ is not

always true. AG

[1]

Chapter 8 Solution

Exercise 31

1. $\because L_1$ and L_2 are perpendicular.

$\therefore \begin{pmatrix} k-1 \\ 20 \\ -10 \end{pmatrix} \cdot \begin{pmatrix} k+2 \\ k-2 \\ k \end{pmatrix} = 0$ (M1) for setting equation

$(k-1)(k+2)+(20)(k-2)+(-10)(k)=0$ (A1) for correct approach

$k^2+k-2+20k-40-10k=0$

$k^2+11k-42=0$

$(k+14)(k-3)=0$

$k=-14$ or $k=3$ A2

[4]

2. $\because L_1$ and L_2 are not perpendicular.

$\therefore \begin{pmatrix} -4k \\ k \\ 0 \end{pmatrix} \cdot \begin{pmatrix} k \\ 1 \\ 1 \end{pmatrix} \neq 0$ (M1) for setting equation

$(-4k)(k)+(k)(1)+(0)(1)\neq 0$ (A1) for correct approach

$-4k^2+k\neq 0$

$4k^2-k\neq 0$

$k(4k-1)\neq 0$

$\therefore k\neq 0$ and $k\neq \frac{1}{4}$ A2

[4]

3. (a) $L_1: \begin{cases} x = 7 + 4s \\ y = 5 + 3s \\ z = -s \end{cases}$, $L_2: \begin{cases} x = 15 + 6t \\ y = -8 - 5t \\ z = 1 \end{cases}$ (M1) for valid approach

$-s = 1$

$s = -1$ A1

[2]

(b) $\begin{cases} x = 7 + 4(-1) = 3 \\ y = 5 + 3(-1) = 2 \\ z = -(-1) = 1 \end{cases}$ (M1) for substitution

Thus, the coordinates of P are $(3, 2, 1)$. A2

[3]

4. (a) $L_1: \begin{cases} x = k + 3s \\ y = -5 - 4s \\ z = -4 - 3s \end{cases}$, $L_2: \begin{cases} x = 5 + t \\ y = 3 + 2t \\ z = 2 + t \end{cases}$ (M1) for valid approach

$-4 - 3s = 2 + t$

$t = -6 - 3s$

$-5 - 4s = 3 + 2t$

$\therefore -5 - 4s = 3 + 2(-6 - 3s)$ (M1) for substitution

$-5 - 4s = -9 - 6s$

$2s = -4$

$s = -2$ A1

[3]

(b) $t = -6 - 3(-2)$

$t = 0$ (A1) for correct value

$k + 3s = 5 + t$

$\therefore k + 3(-2) = 5 + 0$

$k = 11$ A1

[2]

Exercise 32

1. (a) $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} + t\begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$ A2

[2]

(b) (i) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -5 \\ -5 \end{pmatrix}$ A1

(ii) $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -5 \end{pmatrix}$ A2

[3]

2. (a) (i) $\overrightarrow{AB} = \begin{pmatrix} -3 \\ -12 \\ -2 \end{pmatrix}$ A1

(ii) $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix} + t\begin{pmatrix} -3 \\ -12 \\ -2 \end{pmatrix}$ A2

[3]

(b) $BC \perp AB$

$\therefore \overrightarrow{BC} \cdot \overrightarrow{AB} = 0$ (M1) for setting equation

$(k+2)(-3)+(8)(-12)+(-1)(-2)=0$ (A1) for correct approach

$-3k-6-96+2=0$

$-3k=100$

$k=-\frac{100}{3}$ A1

[3]

3. (a) $\mathbf{v} = \begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix} \times \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$ (M1) for valid approach

$\mathbf{v} = \begin{pmatrix} (4)(1) - (2)(-3) \\ (2)(0) - (5)(1) \\ (5)(-3) - (4)(0) \end{pmatrix}$ (A1) for substitution

$\mathbf{v} = \begin{pmatrix} 10 \\ -5 \\ -15 \end{pmatrix}$ A1

[3]

(b) $\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} + s \begin{pmatrix} 10 \\ -5 \\ -15 \end{pmatrix}$ A2

[2]

4. (a) (i) $\vec{AB} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ A1

(ii) $\vec{AC} = \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix}$ A1

[2]

(b) $\vec{AB} \times \vec{AC} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ -3 \end{pmatrix}$

$\vec{AB} \times \vec{AC} = \begin{pmatrix} (3)(-3) - (2)(-2) \\ (2)(2) - (1)(-3) \\ (1)(-2) - (3)(2) \end{pmatrix}$ (A1) for substitution

$\vec{AB} \times \vec{AC} = \begin{pmatrix} -5 \\ 7 \\ -8 \end{pmatrix}$ A1

[2]

(c) $\mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix} + t \begin{pmatrix} -5 \\ 7 \\ -8 \end{pmatrix}$ A2

[2]

Exercise 33

1. (a) $\mathbf{a}+3\mathbf{b}=\begin{pmatrix}5\\12\\0\end{pmatrix}$ A1

[1]

(b) The required component

$=\dfrac{\mathbf{b}\cdot(\mathbf{a}+3\mathbf{b})}{|\mathbf{a}+3\mathbf{b}|}$ (M1) for valid approach

$=\dfrac{(0)(5)+(3)(12)+(0)(0)}{\sqrt{5^2+12^2+0^2}}$ (A1) for substitution

$=\dfrac{36}{13}$ A1

[3]

2. (a) $3\mathbf{a}-2\mathbf{b}=\begin{pmatrix}1\\8\\19\end{pmatrix}$ A1

[1]

(b) The required component

$=\dfrac{|(3\mathbf{a}-2\mathbf{b})\times\mathbf{a}|}{|\mathbf{a}|}$ (M1) for valid approach

$=\dfrac{\left|\begin{pmatrix}(8)(3)-(19)(2)\\(19)(1)-(1)(3)\\(1)(2)-(8)(1)\end{pmatrix}\right|}{\sqrt{1^2+2^2+3^2}}$ (A1) for substitution

$=\dfrac{\sqrt{(-14)^2+16^2+(-6)^2}}{\sqrt{1^2+2^2+3^2}}$

$=5.903993806$

$=5.90$ A1

[3]

3. (a) The required component

$$= \frac{\vec{OA} \cdot \vec{OB}}{\left|\vec{OB}\right|}$$ (M1) for valid approach

$$= \frac{(2)(3)+(6)(4)+(3)(-2)}{\sqrt{3^2+4^2+(-2)^2}}$$ (A1) for substitution

$= 4.456688116$

$= 4.46$ A1

[3]

(b) (i) $\left|\vec{OA}\right| = \sqrt{2^2+6^2+3^2}$ (A1) for substitution

$\left|\vec{OA}\right| = 7$ A1

(ii) $\cos A\hat{O}B = \frac{4.456688116}{7}$ (M1) for valid approach

$A\hat{O}B = 50.45606441^\circ$

$A\hat{O}B = 50.5^\circ$ A1

[4]

4. (a) The required component

$= \dfrac{\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|}{\left|\overrightarrow{OB}\right|}$ (M1) for valid approach

$= \dfrac{\left|\begin{pmatrix} (12)(1)-(4)(3) \\ (4)(5)-(3)(1) \\ (3)(3)-(12)(5) \end{pmatrix}\right|}{\sqrt{5^2+3^2+1^2}}$ (A1) for substitution

$= \dfrac{\sqrt{0^2+17^2+(-51)^2}}{\sqrt{5^2+3^2+1^2}}$

$= 9.086882225$

$= 9.09$ A1

[3]

(b) (i) $\left|\overrightarrow{OA}\right| = \sqrt{3^2+12^2+4^2}$ (A1) for substitution

$\left|\overrightarrow{OA}\right| = 13$ A1

(ii) $\cos O\hat{A}C = \dfrac{9.086882225}{13}$ (M1) for valid approach

$O\hat{A}C = 45.65389703^\circ$

$O\hat{A}C = 45.7^\circ$ A1

[4]

Exercise 34

1. (a) $\mathbf{r}=\begin{pmatrix}10\\0\end{pmatrix}+t\begin{pmatrix}1\\2\end{pmatrix}$ A2

[2]

(b) $\mathbf{s}_A=\begin{pmatrix}10+5\\2(5)\end{pmatrix}$ (M1) for substitution

$\mathbf{s}_A=\begin{pmatrix}15\\10\end{pmatrix}$

Thus, the required coordinates are $(15,10)$. A1

[2]

(c) (i) $\begin{pmatrix}15\\10\end{pmatrix}+7\begin{pmatrix}-4\\-2\end{pmatrix}=\begin{pmatrix}-13\\-4\end{pmatrix}$ (M1) for valid approach

Thus, the required coordinates are $(-13,-4)$. A1

(ii) $\begin{pmatrix}15\\10\end{pmatrix}+15\begin{pmatrix}-4\\-2\end{pmatrix}=\begin{pmatrix}-45\\-20\end{pmatrix}$ (M1) for valid approach

Thus, the required coordinates are $(-45,-20)$. A1

[4]

(d) $\begin{pmatrix}-45\\-20\end{pmatrix}-20\begin{pmatrix}-1\\-1\end{pmatrix}=\begin{pmatrix}-25\\0\end{pmatrix}$ (M1) for valid approach

Thus, the required coordinates are $(-25,0)$. A1

[2]

(e) $\mathbf{r}=\begin{pmatrix}-25\\0\end{pmatrix}+t\begin{pmatrix}-1\\-1\end{pmatrix}$ A2

[2]

(f) The displacement vector of B at $t=12$

$=\begin{pmatrix}-25\\0\end{pmatrix}+12\begin{pmatrix}-1\\-1\end{pmatrix}$ (M1) for substitution

$=\begin{pmatrix}-37\\-12\end{pmatrix}$

The required distance

$=\sqrt{(-37-(-13))^2+(-12-(-4))^2}$ (A1) for substitution

$=25.29822128$

$=25.3$ A1

[3]

2. (a) The required distance

$=\sqrt{(15-3)^2+(9-4)^2}$ (A1) for substitution

$=13$ A1

[2]

(b) $\mathbf{r}=\begin{pmatrix}3\\4\end{pmatrix}+t\begin{pmatrix}12\\5\end{pmatrix}$ A2

[2]

(c) The velocity vector

$=\begin{pmatrix}15\\15\end{pmatrix}-\begin{pmatrix}15\\9\end{pmatrix}$ (M1) for valid approach

$=\begin{pmatrix}0\\6\end{pmatrix}$ A1

[2]

(d) $\begin{pmatrix}15\\9\end{pmatrix}+x\begin{pmatrix}3\\-3\end{pmatrix}=\begin{pmatrix}21\\3\end{pmatrix}$ (M1) for valid approach

$15+3x=21$

$3x=6$

$x=2$ A1

[2]

(e) $\cos\theta=\dfrac{(3)(0)+(-3)(6)}{(\sqrt{3^2+(-3)^2})(\sqrt{0^2+6^2})}$ M1A1

$\cos\theta=-0.707106781$

$\theta=135^\circ$

Thus, the required angle is 135°. A1

[3]

(f) $9+6t=24$ (M1) for valid approach

$6t=15$

$t=2.5$ (A1) for correct value

The amount of time needed

$=2.5+1$

$=3.5\text{ s}$ A1

[3]

3. (a) The velocity vector of A

$$=\frac{1}{2}\left(\begin{pmatrix}0\\20\\0\end{pmatrix}-\begin{pmatrix}-800\\160\\8\end{pmatrix}\right)$$ (M1) for valid approach

$$=\begin{pmatrix}400\\-70\\-4\end{pmatrix}\text{kmh}^{-1}$$ A1

[2]

(b) The speed of A

$$=\sqrt{400^2+(-70)^2+(-4)^2}$$ (A1) for substitution

$$=406.0985102\text{ kmh}^{-1}$$

$$=406\text{ kmh}^{-1}$$ A1

[2]

(c) $\mathbf{r}=\begin{pmatrix}-800\\160\\8\end{pmatrix}+t\begin{pmatrix}400\\-70\\-4\end{pmatrix}$ A2

[2]

(d) $15-5t=0$ (M1) for setting equation

$15=5t$

$t=3$

Thus, B lands on the ground at 11:00. A1

[2]

(e) $\mathbf{r}=\begin{pmatrix}-880\\-180\\15\end{pmatrix}+3\begin{pmatrix}300\\60\\-5\end{pmatrix}$ (M1) for valid approach

$\mathbf{r}=\begin{pmatrix}20\\0\\0\end{pmatrix}$

Thus, the coordinates of B are $(20, 0, 0)$. A1

[2]

(f) $\begin{pmatrix}5s^2-2s\\4s-10s^2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ (M1) for setting equation

$5s^2-2s=0$

$s(5s-2)=0$

$s=0$ (*Rejected*) or $s=0.4$ (A1) for correct value

Thus, the car will stop again at 10:24. A1

[3]

4. (a) $\mathbf{r} = \begin{pmatrix} 20 \\ -10 \\ 0 \end{pmatrix} + t\begin{pmatrix} 5 \\ 5 \\ 0 \end{pmatrix}$ A2

[2]

(b) $40 = 20 + 5p$ (M1) for setting equation

$20 = 5p$

$p = 4$ A1

[2]

(c) The velocity vector of B

$= \frac{1}{4}\left(\begin{pmatrix} 40 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 20 \\ 20 \\ 20 \end{pmatrix}\right)$ (M1) for valid approach

$= \begin{pmatrix} 5 \\ -5 \\ -5 \end{pmatrix} \text{s}^{-1}$ A1

[2]

(d) $\mathbf{r} = \begin{pmatrix} 20 \\ 20 \\ 20 \end{pmatrix} + t\begin{pmatrix} 5 \\ -5 \\ -5 \end{pmatrix}$ A2

[2]

(e) $\mathbf{r}_A = \begin{pmatrix} 20+5t \\ -10+5t \\ 0 \end{pmatrix}$, $\mathbf{r}_B = \begin{pmatrix} 20+5t \\ 20-5t \\ 20-5t \end{pmatrix}$ (A1) for correct values

$\mathbf{r}_A - \mathbf{r}_B = \begin{pmatrix} 20+5t \\ -10+5t \\ 0 \end{pmatrix} - \begin{pmatrix} 20+5t \\ 20-5t \\ 20-5t \end{pmatrix}$

$\mathbf{r}_A - \mathbf{r}_B = \begin{pmatrix} 0 \\ 10t-30 \\ 5t-20 \end{pmatrix}$ (A1) for correct value

$|\mathbf{r}_A - \mathbf{r}_B| = \sqrt{0^2 + (10t-30)^2 + (5t-20)^2}$ (A1) for correct approach

By considering the graph of

$y = \sqrt{(10t-30)^2 + (5t-20)^2}$, the minimum point is

$(3.2000024, 4.472136)$.

Thus, the shortest distance is 4.47. A1

[4]

(f) 3.20 seconds after the start of the game A1

[1]

Exercise 35

1. (a) (i) $L_1:\begin{cases}x=15+6t\\ y=11+3t\\ z=6+2t\end{cases},\ L_2:\begin{cases}x=-3s\\ y=7+2s\\ z=-4-6s\end{cases}$ (M1) for valid approach

$15+6t=-3s$

$s=-5-2t$

$11+3t=7+2s$

$\therefore 11+3t=7+2(-5-2t)$ (M1) for substitution

$11+3t=-3-4t$

$7t=-14$

$t=-2$ A1

(ii) $\therefore \begin{cases}x=15+6(-2)=3\\ y=11+3(-2)=5\\ z=6+2(-2)=2\end{cases}$ (M1) for substitution

Thus, the coordinates of C are $(3, 5, 2)$. A1

[5]

(b) (i) $\overrightarrow{CA}=12\mathbf{i}+6\mathbf{j}+4\mathbf{k}$ A1

(ii) $\overrightarrow{CB}=-3\mathbf{i}+2\mathbf{j}-6\mathbf{k}$ A1

(iii) The area of the triangle ABC

$=\frac{1}{2}\left|\overrightarrow{CA}\times\overrightarrow{CB}\right|$ (M1) for valid approach

$=\frac{1}{2}\left|(12\mathbf{i}+6\mathbf{j}+4\mathbf{k})\times(-3\mathbf{i}+2\mathbf{j}-6\mathbf{k})\right|$

$=\frac{1}{2}\left|\begin{pmatrix}(6)(-6)-(4)(2)\\(4)(-3)-(12)(-6)\\(12)(2)-(6)(-3)\end{pmatrix}\right|$

$=\frac{1}{2}\left|-44\mathbf{i}+60\mathbf{j}+42\mathbf{k}\right|$

$=\frac{1}{2}\sqrt{(-44)^2+60^2+42^2}$ A1

$=42.72001873$

$=42.7$ A1

[5]

(c) The vector equation of the line L_3:

$$\mathbf{r} = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix} + u \begin{pmatrix} -44 \\ 60 \\ 42 \end{pmatrix}$$ A2

[2]

(d) $3 - 44u = 73$

$-44u = 70$

$u = -\frac{35}{22}$ (A1) for correct value

$d = 2 + 42\left(-\frac{35}{22}\right)$

$d = -\frac{713}{11}$ A1

[2]

(e) (i) $\vec{\text{CD}} = 70\mathbf{i} - 100\mathbf{j} - \frac{735}{11}\mathbf{k}$ A1

(ii) The volume of the pyramid ABCD

$= \frac{1}{3}(42.72001873)$

$\left(\sqrt{70^2 + (-100)^2 + \left(-\frac{735}{11}\right)^2}\right)$ M1A1

$= 1981.596486$

$= 1980$ A1

[4]

2. (a) $L_1:\begin{cases} x=8+2t \\ y=8 \\ z=7 \end{cases}$, $L_2:\begin{cases} x=6+s \\ y=8+2\sqrt{3}+\sqrt{3}s \\ z=7 \end{cases}$ M1

$8=8+2\sqrt{3}+\sqrt{3}s$

$-2\sqrt{3}=\sqrt{3}s$

$s=-2$ A1

$x=6+(-2)$ M1

$x=4$

Thus, the coordinates of C are $(4,8,7)$. AG

[3]

(b) $(\mathbf{i}+\sqrt{3}\mathbf{j})\cdot\mathbf{j}=\left|\mathbf{i}+\sqrt{3}\mathbf{j}\right|\left|\mathbf{j}\right|\cos\theta$ (M1) for valid approach

$(1)(0)+(\sqrt{3})(1)=(\sqrt{1^2+(\sqrt{3})^2})(1)\cos\theta$ (A1) for correct approach

$\sqrt{3}=2\cos\theta$

$\cos\theta=\dfrac{\sqrt{3}}{2}$

$\theta=30^\circ$ A1

[3]

(c) (i) $\overrightarrow{\mathrm{CA}}=2\mathbf{i}$ A1

(ii) $\overrightarrow{\mathrm{CB}}=\mathbf{i}+\sqrt{3}\mathbf{j}$ A1

(iii) $\overrightarrow{\mathrm{CA}}\cdot\overrightarrow{\mathrm{CB}}=\left|\overrightarrow{\mathrm{CA}}\right|\left|\overrightarrow{\mathrm{CB}}\right|\cos\mathrm{A\hat{C}B}$ (M1) for valid approach

$2\mathbf{i}\cdot(\mathbf{i}+\sqrt{3}\mathbf{j})=\left|2\mathbf{i}\right|\left|\mathbf{i}+\sqrt{3}\mathbf{j}\right|\cos\mathrm{A\hat{C}B}$ (A1) for substitution

$(2)(1)+(0)(\sqrt{3})=(2)(\sqrt{1^2+(\sqrt{3})^2})\cos\mathrm{A\hat{C}B}$

$2=4\cos\mathrm{A\hat{C}B}$

$\cos\mathrm{A\hat{C}B}=\dfrac{1}{2}$

$\mathrm{A\hat{C}B}=60^\circ$ A1

[5]

(d) $\text{CA} = \text{CB} = 2$

The area of the triangle ABC

$= \frac{1}{2}(\text{CA})(\text{CB})\sin \text{A}\hat{\text{C}}\text{B}$ (M1) for valid approach

$= \frac{1}{2}(2)(2)\sin 60^\circ$ (A1) for substitution

$= 1.732050808$

$= 1.73$ A1

[3]

(e) (i) The triangle ABC is an equilateral triangle.

$\therefore 2(1.732050808) + 3(2h) = 2(30 + \sqrt{3})$ M1A1

$6h = 60$

$h = 10$ A1

(ii) The volume of the prism ABCFED

$= (1.732050808)(10)$

$= 17.32050808$

$= 17.3$ A1

[4]

3. (a) (i) $L_1 : \begin{cases} x = 14 - 5t \\ y = 18 - 6t \\ z = 8 - 2t \end{cases}$ M1

$14 - 5t = 8 - 2t$ M1A1

$6 = 3t$

$t = 2$ AG

(ii) $\begin{cases} x = 14 - 5(2) = 4 \\ y = 18 - 6(2) = 6 \\ z = 8 - 2(2) = 4 \end{cases}$ (M1) for substitution

Thus, the coordinates of P are $(4, 6, 4)$. A1

[5]

(b) $a = 3$ A1

[1]

(c) (i) $\overrightarrow{OR} = 3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ A1

(ii) $\overrightarrow{OQ} = \overrightarrow{OR} + \overrightarrow{RQ}$

$\therefore \overrightarrow{OQ} = (3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}) + (-4\mathbf{i} - 4\mathbf{j} - \mathbf{k})$ (M1) for valid approach

$\overrightarrow{OQ} = -\mathbf{i} + 2\mathbf{k}$

Thus, the coordinates of Q are $(-1, 0, 2)$. A1

[3]

(d) (i) $\overrightarrow{PQ} = -5\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$ A1

(ii) $\overrightarrow{PT} = -30\mathbf{i} - 36\mathbf{j} - 12\mathbf{k}$ A1

(iii) $\overrightarrow{PR} = -\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ A1

(iv) $\overrightarrow{PS} = -14\mathbf{i} - 28\mathbf{j} - 14\mathbf{k}$ A1

(v) The area of the quadrilateral QRST

= The area of PST − The area of PQR (M1) for valid approach

$= \frac{1}{2}\left|\overrightarrow{PS} \times \overrightarrow{PT}\right| - \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|$

$= \frac{1}{2}\left|(-14\mathbf{i} - 28\mathbf{j} - 14\mathbf{k}) \times (-30\mathbf{i} - 36\mathbf{j} - 12\mathbf{k})\right|$

$- \frac{1}{2}\left|(-5\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}) \times (-\mathbf{i} - 2\mathbf{j} - \mathbf{k})\right|$ (A1) for substitution

$= \frac{1}{2}\left|\begin{pmatrix} (-28)(-12) - (-14)(-36) \\ (-14)(-30) - (-14)(-12) \\ (-14)(-36) - (-28)(-30) \end{pmatrix}\right|$

$- \frac{1}{2}\left|\begin{pmatrix} (-6)(-1) - (-2)(-2) \\ (-2)(-1) - (-5)(-1) \\ (-5)(-2) - (-6)(-1) \end{pmatrix}\right|$

$= \frac{1}{2}\left|-168\mathbf{i} + 252\mathbf{j} - 336\mathbf{k}\right| - \frac{1}{2}\left|2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}\right|$ A1

$= \frac{1}{2}\sqrt{(-168)^2 + 252^2 + (-336)^2}$

$- \frac{1}{2}\sqrt{2^2 + (-3)^2 + 4^2}$

$= 223.4843395$

$= 223$ A1

[8]

4. (a) (i) $\overrightarrow{BD} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$

$\overrightarrow{BD} = \begin{pmatrix} -3 \\ -3 \\ -3 \end{pmatrix}$ A1

The vector equation of BD:

$\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ -3 \\ -3 \end{pmatrix}$ A1

(ii) $\begin{cases} x = 3 - 3t \\ y = -3t \\ z = 3 - 3t \end{cases}$ A1

$\overrightarrow{AE} = \begin{pmatrix} 3-3t \\ -3t \\ 3-3t \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

$\overrightarrow{AE} = -3t\mathbf{i} - 3t\mathbf{j} + (3-3t)\mathbf{k}$ A1

(iii) $\overrightarrow{AE} \cdot \overrightarrow{BD} = 0$

$\therefore (-3t)(-3) + (-3t)(-3) + (3-3t)(-3) = 0$ M1

$9t + 9t - 9 + 9t = 0$

$27t = 9$

$t = \frac{1}{3}$ A1

$\therefore \begin{cases} x = 3 - 3\left(\frac{1}{3}\right) = 2 \\ y = -3\left(\frac{1}{3}\right) = -1 \\ z = 3 - 3\left(\frac{1}{3}\right) = 2 \end{cases}$ M1

Therefore, the coordinates of E are $(2, -1, 2)$. AG

[7]

(b) (i) $\overrightarrow{BA} = -3\mathbf{k}$, $\overrightarrow{BC} = -3\mathbf{i}$ A2

(ii) $\mathbf{n}_1 = \overrightarrow{BA} \times \overrightarrow{BD}$

$\mathbf{n}_1 = -3\mathbf{k} \times (-3\mathbf{i} - 3\mathbf{j} - 3\mathbf{k})$

$\mathbf{n}_1 = \begin{pmatrix} (0)(-3) - (-3)(-3) \\ (-3)(-3) - (0)(-3) \\ (0)(-3) - (0)(-3) \end{pmatrix}$ (A1) for substitution

$\mathbf{n}_1 = -9\mathbf{i} + 9\mathbf{j}$ A1

(iii) $\mathbf{n}_2 = \overrightarrow{BC} \times \overrightarrow{BD}$

$\mathbf{n}_2 = -3\mathbf{i} \times (-3\mathbf{i} - 3\mathbf{j} - 3\mathbf{k})$

$\mathbf{n}_2 = \begin{pmatrix} (0)(-3) - (0)(-3) \\ (0)(-3) - (-3)(-3) \\ (-3)(-3) - (0)(-3) \end{pmatrix}$ (A1) for substitution

$\mathbf{n}_2 = -9\mathbf{j} + 9\mathbf{k}$ A1

(iv) $\mathbf{n}_1 \cdot \mathbf{n}_2 = |\mathbf{n}_1||\mathbf{n}_2|\cos\theta$ (M1) for valid approach

$(-9\mathbf{i} + 9\mathbf{j}) \cdot (-9\mathbf{j} + 9\mathbf{k})$

$= |-9\mathbf{i} + 9\mathbf{j}||-9\mathbf{j} + 9\mathbf{k}|\cos\theta$

$(-9)(0) + (9)(-9) + (0)(9)$

$= (\sqrt{(-9)^2 + 9^2})(\sqrt{(-9)^2 + 9^2})\cos\theta$ A1

$-81 = 162\cos\theta$

$\cos\theta = -\frac{1}{2}$

$\theta = 120^\circ$

Therefore, the required acute angle is 60°. A1

[9]

(c) The area of $OABC$

$= (OA)(OC)$

$= (3)(3)$

$= 9$ (A1) for correct value

$\therefore \frac{1}{3}(9)(OF) = 15$ (M1) for setting equation

$3OF = 15$

$OF = 5$ A1

Thus, the possible coordinates of F are $(0, 5, 0)$ and $(0, -5, 0)$. A1

[4]

Chapter 9 Solution

Exercise 36

1.	(a)	3	A1	
				[1]
	(b)	$\mathbf{M}=\begin{pmatrix}0&1&1&1\\1&0&1&1\\1&1&0&1\\1&1&1&0\end{pmatrix}$	A2	
				[2]
	(c)	$\mathbf{M}^3=\begin{pmatrix}6&7&7&7\\7&6&7&7\\7&7&6&7\\7&7&7&6\end{pmatrix}$	(M1) for valid approach	
		Thus, the number of walks of length 3 from A to itself is 6.	A1	
				[2]
2.	(a)	5	A1	
				[1]
	(b)	$\mathbf{M}=\begin{pmatrix}0&1&0&2\\1&2&1&1\\0&1&0&1\\2&1&1&0\end{pmatrix}$	A2	
				[2]
	(c)	$\mathbf{M}^4=\begin{pmatrix}51&62&34&34\\62&99&44&72\\34&44&23&26\\34&72&26&63\end{pmatrix}$	(M1) for valid approach	
		The required total number of walks		
		$=51+99+23+63$	(A1) for correct approach	
		$=236$	A1	
				[3]

3. (a) (i) 2 A1

(ii) 1 A1

[2]

(b) $$\mathbf{M} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$ A3

[3]

(c) $$\mathbf{M}^{12} = \begin{pmatrix} 1 & 3 & 1 & 3 & 1 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 3 & 1 & 3 & 1 \\ 3 & 4 & 3 & 2 & 3 \\ 2 & 3 & 1 & 1 & 1 \end{pmatrix}$$ (M1) for valid approach

Thus, the total number of walks of length 12 from D to B is 4. A1

[2]

4. (a) (i) 1 A1

(ii) 2 A1

[2]

(b) $$\mathbf{M} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ A3

[3]

(c) $$\mathbf{M} + \mathbf{M}^2 + \mathbf{M}^3 = \begin{pmatrix} 1 & 1 & 2 & 2 & 2 & 1 \\ 1 & 0 & 3 & 3 & 2 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$ (M1) for valid approach

Thus, the total number of walks of length at most 3 from A to E is 2. A1

[2]

Exercise 37

1.	(a)	BC, DE	A1	
				[1]
	(b)	For any two edges correct	A1	
		For all edges correct	A1	
		1. Choose BC of weight 12		
		2. Choose DE of weight 12		
		3. Choose AB of weight 16		
		4. Choose BD of weight 18		
		Thus, the minimum spanning tree is a tree containing BC, DE, AB and BD.	A1	
				[3]
	(c)	58	A1	
				[1]
2.	(a)	CD, EG	A1	
				[1]
	(b)	44	A1	
				[1]
	(c)	For any three edges correct	A1	
		For all edges correct	A1	
		1. Choose CD of cost 30		
		2. Choose EG of cost 30		
		3. Choose AB of cost 33		
		4. Choose AF of cost 36		
		5. Choose CE of cost 39		
		6. Choose AC of cost 42		
		Thus, the minimum spanning tree is a tree containing CD, EG, AB, AF, CE and AC.	A1	
				[3]
	(d)	210	A1	
				[1]

3. (a) $\mathbf{M} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$ A2

[2]

(b) AB A1

[1]

(c) For any two edges correct A1

For all edges correct A1

1. Choose AB of weight 10
2. Choose EF of weight 15
3. Choose DE of weight 20
4. Choose CD of weight 25
5. Choose BE of weight 35

Thus, the minimum spanning tree is a tree containing AB, EF, DE, CD and BE. A1

[3]

(d) 105 A1

[1]

4. (a) $\mathbf{M} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$ A2

[2]

(b) AE A1

[1]

(c) For any two edges correct A1
For any four edges correct A1
For all edges correct A1

1. Choose AE of weight 30
2. Choose CD of weight 35
3. Choose FG of weight 35
4. Choose AB of weight 40
5. Choose DE of weight 50
6. Choose EF of weight 65
7. Choose EH of weight 80

Thus, the minimum spanning tree is a tree containing AE, CD, FG, AB, DE, EF and EH. A1

[4]

(d) 335 A1

[1]

Exercise 38

1.	(a)	AB	A1	
				[1]
	(b)	For any two edges correct	A1	
		For all edges correct	A1	
		1. Choose AE of weight 31		
		2. Choose DE of weight 14		
		3. Choose BD of weight 12		
		4. Choose CD of weight 31		
		Thus, the minimum spanning tree is a tree containing AE, DE, BD and CD.	A1	
				[3]
	(c)	88	A1	
				[1]
2.	(a)	37	A1	
				[1]
	(b)	For any three edges correct	A1	
		For all edges correct	A1	
		1. Choose BE of cost 29		
		2. Choose CE of cost 30		
		3. Choose AB of cost 38		
		4. Choose AF of cost 26		
		5. Choose FG of cost 28		
		6. Choose CD of cost 42		
		Thus, the minimum spanning tree is a tree containing BE, CE, AB, AF, FG and CD.	A1	
				[3]
	(c)	193	A1	
				[1]

3.	(a)	$\mathbf{M} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$	A2	
				[2]
	(b)	For any two edges correct	A1	
		For all edges correct	A1	
		1. Choose EF of weight 23		
		2. Choose CE of weight 29		
		3. Choose CD of weight 19		
		4. Choose AF of weight 37		
		5. Choose AB of weight 31		
		Thus, the minimum spanning tree is a tree containing EF, CE, CD, AF and AB.	A1	
				[3]
	(c)	139	A1	
				[1]

4. (a) $\mathbf{M} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix}$ A2

[2]

(b) DE A1

[1]

(c) For any three edges correct A1
For any six edges correct A1
For all edges correct A1

1. Choose HI of weight 68
2. Choose FI of weight 74
3. Choose DF of weight 72
4. Choose BD of weight 66
5. Choose FG of weight 76
6. Choose AH of weight 78
7. Choose EF of weight 80
8. Choose BC of weight 86

Thus, the minimum spanning tree is a tree containing HI, FI, DF, BD, FG, AH, EF and BC. A1

[4]

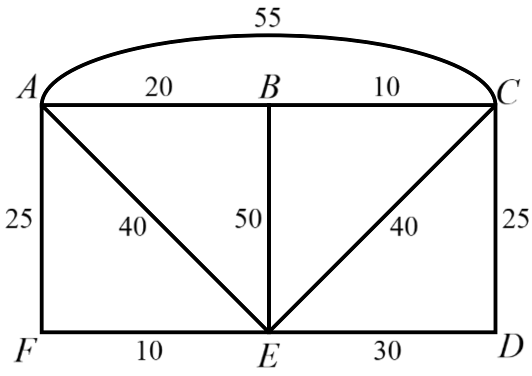
(d) 600 A1

[1]

Exercise 39

1. (a) For correct edges A1
For correct vertices A1
For correct weights A1

[3]



(b) (i) 5 A1

(ii) 2 A1

[2]

(c) $$\mathbf{M} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ A2

[2]

(d) $$\mathbf{M}^4 = \begin{pmatrix} 38 & 30 & 33 & 22 & 40 & 19 \\ 30 & 29 & 30 & 20 & 32 & 20 \\ 33 & 30 & 38 & 19 & 40 & 22 \\ 22 & 20 & 19 & 15 & 21 & 13 \\ 40 & 32 & 40 & 21 & 49 & 21 \\ 19 & 20 & 22 & 13 & 21 & 15 \end{pmatrix}$$ (M1) for valid approach

Thus, the total number of walks of length 4 from C to A is 33. A1

[2]

(e)	For any three edges correct	A1	
	For any six edges correct	A1	
	1. Choose EF of weight 10		
	2. Choose FA of weight 25		
	3. Choose AB of weight 20		
	4. Choose BC of weight 10		
	5. Choose CD of weight 25		
	6. Choose DE of weight 30		
	7. Choose EA of weight 40		
	8. Choose AC of weight 55		
	9. Choose CE of weight 40		
	10. Choose EB of weight 50		
	11. Choose BE of weight 50		
	Thus, a possible route contains EF, FA, AB, BC, CD, DE, EA, AC, CE, EB and BE.	A1	
			[3]
(f)	355	A1	
			[1]

2.	(a)	(i)	6	A1	
		(ii)	2	A1	
		(iii)	5	A1	
					[3]
	(b)	AE		A1	
					[1]
	(c)	For any three edges correct		A1	
		For all edges correct		A1	
		1.	Choose AE of weight 35		
		2.	Choose AF of weight 39		
		3.	Choose GF of weight 47		
		4.	Choose AC of weight 49		
		5.	Choose BC of weight 58		
		6.	Choose AD of weight 68		
		Thus, the minimum spanning tree is a tree containing AE, AF, GF, AC, BC and AD.		A1	
					[3]
	(d)	296		A1	
					[1]
	(e)	For any four edges correct		A1	
		For any eight edges correct		A1	
		1.	Choose FG of weight 47		
		2.	Choose GA of weight 57		
		3.	Choose AB of weight 69		
		4.	Choose BC of weight 58		
		5.	Choose CD of weight 79		
		6.	Choose DA of weight 68		
		7.	Choose AE of weight 35		
		8.	Choose EF of weight 49		
		9.	Choose FA of weight 39		
		10.	Choose AC of weight 49		
		11.	Choose CA of weight 49		
		12.	Choose AF of weight 39		
		Thus, a possible route contains FG, GA, AB, BC, CD, DA, AE, EF, FA, AC, CA and AF.		A1	
					[3]
	(f)	638		A1	
					[1]

3.	(a)	(i)	3	A1	
		(ii)	4	A1	
		(iii)	3	A1	
		(iv)	15 minutes	A1	
					[4]

(b) For any four edges correct A1
For any eight edges correct A1

1. Choose CD of weight 20
2. Choose DE of weight 15
3. Choose EF of weight 5
4. Choose FA of weight 15
5. Choose AB of weight 10
6. Choose BC of weight 5
7. Choose CA of weight 28
8. Choose AG of weight 20
9. Choose GD of weight 20
10. Choose DE of weight 15
11. Choose EG of weight 15
12. Choose GB of weight 10
13. Choose BC of weight 5

Thus, a possible route contains CD, DE, EF, FA, AB, BC, CA, AG, GD, DE, EG, GB and BC. A1

[3]

(c) 183 minutes A1

[1]

(d)	(i)	B	A1
	(ii)	For any four edges correct	A1
		For any eight edges correct	A1
		1. Choose CD of weight 20	
		2. Choose DE of weight 15	
		3. Choose EF of weight 5	
		4. Choose FA of weight 15	
		5. Choose AB of weight 10	
		6. Choose BC of weight 5	
		7. Choose CA of weight 28	
		8. Choose AG of weight 20	
		9. Choose GD of weight 20	
		10. Choose DE of weight 15	
		11. Choose EG of weight 15	
		12. Choose GB of weight 10	
		Thus, a possible route contains CD, DE, EF, FA, AB, BC, CA, AG, GD, DE, EG and GB.	A1
	(iii)	178 minutes	A1

[5]

4.	(a)	(i)	5	A1	
		(ii)	2	A1	
		(iii)	7	A1	
		(iv)	170 seconds	A1	
					[4]

(b) For any five edges correct — A1

For any ten edges correct — A1

1. Choose DE of weight 90
2. Choose EF of weight 75
3. Choose FG of weight 80
4. Choose GH of weight 65
5. Choose HA of weight 90
6. Choose AB of weight 105
7. Choose BC of weight 120
8. Choose CD of weight 90
9. Choose DB of weight 90
10. Choose BI of weight 95
11. Choose ID of weight 85
12. Choose DF of weight 115
13. Choose FI of weight 60
14. Choose IH of weight 75
15. Choose HF of weight 95

Thus, a possible route contains DE, EF, FG, GH, HA, AB, BC, CD, DB, BI, ID, DF, FI, IH and HF. A1

[3]

(c) 1330 seconds — A1

[1]

(d)	(i)	For any five edges correct	A1
		For any ten edges correct	A1
		1. Choose DE of weight 90	
		2. Choose EF of weight 75	
		3. Choose FG of weight 80	
		4. Choose GH of weight 65	
		5. Choose HA of weight 90	
		6. Choose AB of weight 105	
		7. Choose BC of weight 120	
		8. Choose CD of weight 90	
		9. Choose DB of weight 90	
		10. Choose BI of weight 95	
		11. Choose ID of weight 85	
		12. Choose DF of weight 115	
		13. Choose FI of weight 60	
		14. Choose IH of weight 75	
		15. Choose HF of weight 95	
		16. Choose FD of weight 115	
		Thus, a possible route contains DE, EF, FG, GH, HA, AB, BC, CD, DB, BI, ID, DF, FI, IH, HF and FD.	A1
	(ii)	1445 seconds	A1

[4]

Exercise 40

1.	(a)	(i)	4	A1	
		(ii)	A, D	A1	
					[2]
	(b)	BD		A1	
					[1]
	(c)	Eulerian trail exists.		A1	
		As there are only two vertices of odd degrees.		A1	
					[2]
	(d)	(i)	70	A1	
		(ii)	90	A1	
					[2]
	(e)	For any three edges correct		A1	
		For all edges correct		A1	
		1.	Choose BD of weight 23		
		2.	Choose DC of weight 47		
		3.	Choose CE of weight 90		
		4.	Choose EA of weight 31		
		5.	Choose AF of weight 29		
		6.	Choose FB of weight 70		
		Thus, the required upper bound is 290.		A1	
					[3]
	(f)	For any two edges correct		A1	
		For all edges correct		A1	
		1.	Choose AF of weight 29		
		2.	Choose AE of weight 31		
		3.	Choose CD of weight 47		
		4.	Choose DE of weight 59		
		Therefore, the weight of a minimum spanning tree after deleting the vertex B is 166.		A1	
		The required lower bound			
		$=166+23+37$			
		$=226$		A1	
					[4]

2. (a) Eulerian trail does not exist. A1

As there are more than two vertices of odd degrees. A1

[2]

(b) $$\mathbf{M}=\begin{pmatrix}0&1&1&0&1&1\\1&0&1&0&1&0\\1&1&0&1&1&0\\0&0&1&0&1&1\\1&1&1&1&0&1\\1&0&0&1&1&0\end{pmatrix}$$ A2

[2]

(c) $$\mathbf{M}^3=\begin{pmatrix}8&9&12&6&12&10\\9&6&9&6&11&6\\12&9&8&10&12&6\\6&6&10&4&10&8\\12&11&12&10&12&10\\10&6&6&8&10&4\end{pmatrix}$$ (M1) for valid approach

Thus, the total number of walks of length 3 from C to A is 12. A1

[2]

(d) For any two edges correct A1

For all edges correct A1

1. Choose CD of weight 30
2. Choose AB of weight 35
3. Choose AE of weight 40
4. Choose AC of weight 45
5. Choose EF of weight 55

Thus, the minimum spanning tree is a tree containing CD, AB, AE, AC and EF. A1

[3]

(e) 205 A1

[1]

(f) For any three edges correct A1

For all edges correct A1

1. Choose DC of weight 30
2. Choose CA of weight 45
3. Choose AB of weight 35
4. Choose BE of weight 50
5. Choose EF of weight 55
6. Choose FD of weight 75

Thus, the required upper bound is 290. A1

[3]

(g) For any two edges correct A1

For all edges correct A1

1. Choose AB of weight 35
2. Choose AE of weight 40
3. Choose AC of weight 45
4. Choose EF of weight 55

Therefore, the weight of a minimum spanning tree after deleting the vertex D is 175. A1

The required lower bound

$=175+30+50$

$=255$ A1

[4]

3.	(a)	(i)	4	A1	
		(ii)	2	A1	
					[2]
	(b)	For any three edges correct		A1	
		For any six edges correct		A1	
		1.	Choose CD of weight 15		
		2.	Choose DE of weight 12		
		3.	Choose EF of weight 13		
		4.	Choose FA of weight 17		
		5.	Choose AF of weight 28		
		6.	Choose FA of weight 17		
		7.	Choose AB of weight 10		
		8.	Choose BC of weight 11		
		9.	Choose CD of weight 15		
		10.	Choose DC of weight 21		
		Thus, a possible route contains CD, DE, EF, FA, AF, FA, AB, BC, CD and DC.		A1	
					[3]
	(c)	159		A1	
					[1]
	(d)	For any three edges correct		A1	
		For all edges correct		A1	
		1.	Choose AB of weight 10		
		2.	Choose BC of weight 11		
		3.	Choose CD of weight 15		
		4.	Choose DE of weight 12		
		5.	Choose EF of weight 13		
		6.	Choose FA of weight 17		
		Thus, the required upper bound is 78.		A1	
					[3]
	(e)	For any two edges correct		A1	
		For all edges correct		A1	
		1.	Choose BC of weight 11		
		2.	Choose DE of weight 12		
		3.	Choose EF of weight 13		
		4.	Choose CD of weight 15		
		Therefore, the weight of a minimum spanning tree after deleting the vertex A is 51.		A1	
		The required lower bound			
		$= 51 + 10 + 17$			
		$= 78$		A1	
					[4]

4.	(a)	Eulerian circuit does not exist.	A1	
		As not all vertices are of even degree.	A1	
				[2]
	(b)	AG	A1	
				[1]
	(c)	For any three edges correct	A1	
		For all edges correct	A1	
		1. Choose AG of weight 61		
		2. Choose AE of weight 72		
		3. Choose AB of weight 80		
		4. Choose AD of weight 87		
		5. Choose CD of weight 93		
		6. Choose EF of weight 97		
		Thus, the minimum spanning tree is a tree containing AG, AE, AB, AD, CD and EF.	A1	
				[3]
	(d)	490	A1	
				[1]
	(e)	For any three edges correct	A1	
		For all edges correct	A1	
		1. Choose DA of weight 87		
		2. Choose AG of weight 61		
		3. Choose GE of weight 83		
		4. Choose EF of weight 97		
		5. Choose FB of weight 240		
		6. Choose BC of weight 131		
		7. Choose CD of weight 93		
		Thus, the required upper bound is 792.	A1	
				[3]
	(f)	For any two edges correct	A1	
		For all edges correct	A1	
		1. Choose AG of weight 61		
		2. Choose AE of weight 72		
		3. Choose AB of weight 80		
		4. Choose EF of weight 97		
		5. Choose BC of weight 131		
		Therefore, the weight of a minimum spanning tree after deleting the vertex D is 441.	A1	
		The required lower bound		
		$= 441 + 87 + 93$		
		$= 621$	A1	
				[4]

Chapter 10 Solution

Exercise 41

1. (a) $f'(x) = (3)(\cos x) + (3x)(-\sin x)$ (M1) for product rule

$f'(x) = 3(\cos x - x\sin x)$ A1

[2]

(b) (i) The gradient of the tangent

$= f'\left(\frac{3\pi}{2}\right)$

$= 3\left(\cos\frac{3\pi}{2} - \frac{3\pi}{2}\sin\frac{3\pi}{2}\right)$ (M1) for substitution

$= 14.13716694$

$= 14.1$ A1

(ii) The gradient of the normal

$= \frac{-1}{f'\left(\frac{3\pi}{2}\right)}$

$= \frac{-1}{14.13716694}$ (M1) for valid approach

$= -0.0707355303$

$= -0.0707$ A1

[4]

2. (a) $f'(x) = (e^{-3x})(-3)$ (M1) for chain rule

$f'(x) = -3e^{-3x}$ A1

[2]

(b) (i) The gradient of the tangent

$= f'(0.1)$

$= -3e^{-3(0.1)}$ (M1) for substitution

$= -2.222454662$

$= -2.22$ A1

(ii) The gradient of the normal

$= \dfrac{-1}{f'(0.1)}$

$= \dfrac{-1}{-2.222454662}$ (M1) for valid approach

$= 0.4499529359$

$= 0.450$ A1

[4]

3. (a) $f'(x) = (-\sin(x^2))(2x)$ (M1) for chain rule

$f'(x) = -2x\sin(x^2)$ A1

[2]

(b) $f'(a) = -2a$

$\therefore -2a\sin(a^2) = -2a$ (M1) for setting equation

$\sin(a^2) = 1$

$\sin(a^2) - 1 = 0$ (M1) for valid approach

By considering the graph of $y = \sin(a^2) - 1$,

$a = 1.253314137$.

$\therefore a = 1.25$ A1

[3]

4. (a) $g'(x) = (2x)(\ln x) + (x^2)\left(\frac{1}{x}\right)$ (M1) for product rule

$g'(x) = 2x\ln x + x$ A1

[2]

(b) $g'(a) = \sqrt{a}$

$\therefore 2a\ln a + a = \sqrt{a}$ (M1) for setting equation

$2a\ln a + a - \sqrt{a} = 0$ (M1) for valid approach

By considering the graph of $y = 2a\ln a + a - \sqrt{a}$,

$a = 1$. A1

[3]

Exercise 42

1. (a) $f'(x) = (\cos 3x)(3)$ (M1) for chain rule

$f'(x) = 3\cos 3x$ A1

[2]

(b) (i) The gradient of the tangent

$= f'(\pi)$

$= 3\cos 3\pi$ (M1) for substitution

$= -3$ A1

(ii) The equation of the tangent:

$y = -3x + b$ (M1) for setting equation

$0 = -3(\pi) + b$ (M1) for substitution

$b = 3\pi$

$\therefore y = -3x + 3\pi$ A1

[5]

2. (a) $f'(x) = \left(\frac{1}{x^3}\right)(3x^2)$ (M1) for chain rule

$f'(x) = \frac{3}{x}$ A1

[2]

(b) (i) The gradient of the tangent

$= f'(1)$

$= \frac{3}{1}$ (M1) for substitution

$= 3$ A1

(ii) $-\frac{1}{3}$ A1

(iii) The equation of the normal:

$y = -\frac{1}{3}x + b$ (M1) for setting equation

$0 = -\frac{1}{3}(1) + b$ (M1) for substitution

$b = \frac{1}{3}$

$\therefore y = -\frac{1}{3}x + \frac{1}{3}$ A1

[5]

3.	(a)	$f'(x) = (e^{3x})(3)$	(M1) for chain rule
		$f'(x) = 3e^{3x}$	A1
			[2]
	(b)	$3e^{3k}$	A1
			[1]
	(c)	$3k - \frac{1}{e^3}(e^{3k}) - 2 = 0$	(M1) for setting equation
		$3k - e^{3k-3} - 2 = 0$	(M1) for valid approach
		By considering the graph of $y = 3k - e^{3k-3} - 2$,	
		$k = 1$.	A1
			[3]
4.	(a)	$f'(x) = \left(\frac{1}{\sqrt{x}}\right)\left(\frac{1}{2\sqrt{x}}\right)$	(M1) for chain rule
		$f'(x) = \frac{1}{2x}$	A1
			[2]
	(b)	$\frac{1}{4}$	A1
			[1]
	(c)	The y-intercept of the normal	
		$= \ln\sqrt{2} - 2m$	(M1) for valid approach
		$= \ln\sqrt{2} - 2\left(-1 \div \frac{1}{4}\right)$	(A1) for substitution
		$= \ln\sqrt{2} + 8$	A1
			[3]

Exercise 43

1. (a) $f'(x) = 0 + 9(1) + 3(2x) - 3x^2$ (A1) for correct approach

$f'(x) = 9 + 6x - 3x^2$ A1

[2]

(b) $f'(x) = 0$

$9 + 6x - 3x^2 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$ (A1) for factorization

$x = -1$ or $x = 3$ A1

[2]

(c) (i) $f''(x) = 6 - 6x$ A1

(ii) $f''(-1) = 6 - 6(-1)$

$f''(-1) = 12 > 0$ R1

Therefore, f attains its local minimum at $x = -1$.

Thus, the x-coordinate of the local minimum of f is -1. A1

(iii) -4 A1

[4]

2. (a) $f'(x) = \left(e^{-\frac{1}{2}x^2}\right)\left(-\frac{1}{2}\right)(2x)$ (M1) for chain rule

$f'(x) = -xe^{-\frac{1}{2}x^2}$ A1

[2]

(b) $f'(x) = 0$

$-xe^{-\frac{1}{2}x^2} = 0$

$-x = 0$ (M1) for valid approach

$x = 0$ A1

[2]

(c) (i) $f''(0) = (0^2 - 1)e^{-\frac{1}{2}(0)^2}$ M1

$f''(0) = -1 < 0$ R1

Therefore, f attains its local maximum at $x = 0$.

Thus, the x-coordinate of the local maximum of f is 0. AG

(ii) 1 A1

[3]

3. (a) $f'(x) = \dfrac{(x+1)^2(4)-(4x)(2)(x+1)}{(x+1)^4}$ (M1) for quotient rule

$f'(x) = \dfrac{4x+4-8x}{(x+1)^3}$

$f'(x) = \dfrac{4-4x}{(x+1)^3}$ A1

[2]

(b) $f'(x) = 0$

$\dfrac{4-4x}{(x+1)^3} = 0$

$4-4x = 0$ (M1) for valid approach

$4 = 4x$

$x = 1$ A1

[2]

(c) (i) $f''(1) = \dfrac{8(1-2)}{(1+1)^4}$ M1

$f''(1) = -\dfrac{1}{2} < 0$ R1

Therefore, f attains its local maximum at $x = 1$.

Thus, the x-coordinate of the local maximum of f is 1. AG

(iii) 1 A1

[3]

4. (a) $f'(x)=\left(\cos\left(2x-\frac{\pi}{3}\right)\right)(2)-0$ (M1) for chain rule

$f'(x)=2\cos\left(2x-\frac{\pi}{3}\right)$ A1

[2]

(b) $f'(x)=0$

$2\cos\left(2x-\frac{\pi}{3}\right)=0$

By considering the graph of $y=2\cos\left(2x-\frac{\pi}{3}\right)$,

$x=-0.2617993878$ or $x=1.308996939$. (M1) for valid approach

$\therefore x=-0.262$ or $x=1.31$ A1

[2]

(c) (i) $f''(x)=-4\sin\left(2x-\frac{\pi}{3}\right)$ A1

(ii) $f''(-0.2617993878)$

$=-4\sin\left(2(-0.2617993878)-\frac{\pi}{3}\right)$

$=-4\sin\left(2(-0.2617993878)-\frac{\pi}{3}\right)$

$=4>0$ R1

Therefore, f attains its local minimum at $x=-0.2617993878$.

Thus, the x-coordinate of the local minimum of f is -0.262. A1

(iii) -4 A1

[4]

Exercise 44

1. (a) $n(0)=300e^{0.28(0)}$ (M1) for substitution

$n(0)=300$ A1

[2]

(b) (i) $\frac{dn}{dt}=300(e^{0.28t})(0.28)$ (M1) for chain rule

$\frac{dn}{dt}=84e^{0.28t}$ A1

(ii) $\left.\frac{dn}{dt}\right|_{t=6}=450.70671$ per month

$\left.\frac{dn}{dt}\right|_{t=6}=451$ per month A1

[3]

(c) $\left.\frac{dn}{dt}\right|_{t=k}>1000$

$84e^{0.28k}>1000$ (M1) for setting inequality

$84e^{0.28k}-1000>0$ (M1) for valid approach

By considering the graph of $y=84e^{0.28k}-1000$,

$k>8.8462089$.

Thus, the least value of k is 9. A1

[3]

2. (a) $V(8)=\sqrt{100-8^2}$ (M1) for substitution

$V(8)=6\text{ cm}^3$ A1

[2]

(b) (i) $\dfrac{dV}{dt}=\left(\dfrac{1}{2\sqrt{100-t^2}}\right)(-2t)$ (M1) for chain rule

$\dfrac{dV}{dt}=-\dfrac{t}{\sqrt{100-t^2}}$ A1

(ii) $0.75\text{ cm}^3\text{s}^{-1}$ A1

[3]

(c) $V'(k)>-0.9$

$-\dfrac{k}{\sqrt{100-k^2}}>-0.9$ (M1) for setting inequality

$0.9-\dfrac{k}{\sqrt{100-k^2}}>0$ (M1) for valid approach

By considering the graph of $y=0.9-\dfrac{k}{\sqrt{100-k^2}}$,

$k<6.6896473$.

Thus, the greatest value of k is 6. A1

[3]

3. (a) The required number of sheep

$= p(5)$ (A1) for correct approach

$= 250\sin(2(5)+3.9)+750$ (A1) for substitution

$= 993.0018753$

$= 993$ A1

[3]

(b) $\frac{dp}{dt} = 250\cos(2t+3.9)(2)+0$ (M1) for chain rule

$\frac{dp}{dt} = 500\cos(2t+3.9)$ A1

[2]

(c) $\frac{dp}{dt} = 0$ (M1) for setting equation

$500\cos(2t+3.9) = 0$

By considering the graph of $y = 500\cos(2t+3.9)$,

$t = 0.4061945$.

The value of n

$= (0.4061945)(31)$ (A1) for correct approach

$= 12.5920295$

$= 13$ A1

[3]

4. (a) The required number of wolves

$= w(13)$ (A1) for correct approach

$= 145\cos(0.5(13)-5.2)+1020$ (A1) for substitution

$= 1058.78733$

$= 1059$ A1

[3]

(b) $\frac{dw}{dt} = 145(-\sin(0.5t-5.2))(0.5)+0$ (M1) for chain rule

$\frac{dw}{dt} = -72.5\sin(0.5t-5.2)$ A1

[2]

(c) By considering the graph of

$y = -72.5\sin(0.5t-5.2)$, $\frac{dw}{dt}$ attains its maximum

for the first time when $t = 7.2584082$. (A1) for correct value

The value of n

$= (7.2584082)(30)$ (A1) for correct approach

$= 217.752246$

$= 218$ A1

[3]

Exercise 45

1.	(a)	By considering the graph of $y = t^2 \cos t$, the maximum distance $= 5.007148$ cm	(M1) for valid approach	
		$= 5.01$ cm	A1	
				[2]
	(b) (i)	$s'(t) = (2t)(\cos t) + (t^2)(-\sin t)$	(M1) for product rule	
		$s'(t) = 2t\cos t - t^2 \sin t$	A1	
	(ii)	By considering the graph of $y = 2t\cos t - t^2 \sin t$, the particle first changes direction at 1.076874 s.	(M1) for valid approach	
		Thus, the required time is 1.08 s.	A1	
	(iii)	$s''(1.076874) = -3.394272 \text{ cms}^{-2}$		
		$s''(1.076874) = -3.39 \text{ cms}^{-2}$	A1	
				[5]
2.	(a)	By considering the graph of $y = 4t\cos t$, the maximum distance $= 5.653563$ cm	(M1) for valid approach	
		$= 5.65$ cm	A1	
				[2]
	(b) (i)	By considering the graph of $y = 4t\cos t$, the particle first goes back to O at 1.5707963 s.	(M1) for valid approach	
		Thus, the required time is 1.57 s.	A1	
	(ii)	$s'(t) = (4)(\cos t) + (4t)(-\sin t)$	(M1) for product rule	
		$s'(t) = 4\cos t - 4t\sin t$	A1	
	(iii)	$s''(1.5707963) = -7.999997502 \text{ cms}^{-2}$		
		$s''(1.5707963) = -8.00 \text{ cms}^{-2}$	A1	
				[5]

3. (a) $v(t) = s'(t)$ (M1) for valid approach

$v(t) = 1 + (\cos(e^t))(e^t)$ (M1) for chain rule

$v(t) = 1 + e^t \cos(e^t)$ A1

[3]

(b) (i) By considering the graph of $y = 1 + e^t \cos(e^t)$, the particle changes direction for the 4th time at 2.3891023 s. (M1) for valid approach

Thus, the required time is 2.39 s. A1

(ii) $v'(2.3891023) = 117.38686 \text{ cms}^{-2}$

$v'(2.3891023) = 117 \text{ cms}^{-2}$ A1

[3]

4. (a) By considering the graph of $y = e^t \sin t$, the particle changes direction for the 1st time and the 3rd time at 2.3561934 s and 8.6393818 s respectively. (M1) for valid approach

The amount of time

$= (8.6393818 - 2.3561934)$ s (M1) for valid approach

$= 6.2831884$ s

$= 6.28$ s A1

[3]

(b) (i) By considering the graph of $y = e^t \sin t$, the particle is at the maximum distance from O at 8.6393781 s. (M1) for valid approach

Thus, the required time is 8.64 s. A1

(ii) $s'(t) = (e^t)(\sin t) + (e^t)(\cos t)$ (M1) for product rule

$s'(t) = e^t(\sin t + \cos t)$ A1

(iii) $s''(8.6393781) = -7990.034257 \text{ cms}^{-2}$

$s''(8.6393781) = -7990 \text{ cms}^{-2}$ A1

[5]

Exercise 46

1.	(a)	(i)	$\pi - \theta$	A1
		(ii)	$P = \frac{1}{2}(\text{OB})(\text{OC})\sin \text{B}\hat{\text{O}}\text{C}$ $+\frac{1}{2}(\text{OA})(\text{OC})\sin \text{A}\hat{\text{O}}\text{C}$	(M1) for valid approach
			$P = \frac{1}{2}(4)(4)\sin\theta + \frac{1}{2}(4)(4)\sin(\pi - \theta)$	(A1) for substitution
			$P = 8\sin\theta + 8\sin\theta$	(A1) for correct approach
			$P = 16\sin\theta$	A1
				[5]
	(b)	(i)	$\frac{dP}{d\theta} = 16\cos\theta$	A1
		(ii)	$\frac{dP}{d\theta} = 0$	
			$\therefore 16\cos\theta = 0$	
			By considering the graph of $y = 16\cos\theta$,	
			$\theta = 1.570796327$.	(M1) for valid approach
			$\therefore \theta = 1.57$ rad	A1
		(iii)	$\frac{d^2P}{d\theta^2} = -16\sin\theta$	(A1) for correct approach
			$\left.\frac{d^2P}{d\theta^2}\right\|_{\theta=1.570796327} = -16\sin 1.570796327$	
			$= -16 < 0$	R1
			Thus, P attains its maximum at	
			$\theta = 1.57$ rad.	A1
		(iv)	The maximum value of P	
			$= 16\sin 1.570796327$	(M1) for substitution
			$= 16$	A1
				[8]
	(c)		$\theta = 0$, $\theta = \pi$	A2
				[2]

2. (a) (i) $\text{CD} = 10\sin\theta$ A1

$\text{OD} = 10\cos\theta$ A1

(ii) The area of the inscribed rectangle

$= 8(\text{Area of } \Delta\text{OCD})$ (M1) for valid approach

$= 8\left(\dfrac{(10\sin\theta)(10\cos\theta)}{2}\right)$

$= 400\sin\theta\cos\theta$ A1

(iii) $P = \pi(10)^2 - 400\sin\theta\cos\theta$ (M1) for valid approach

$P = 100\pi - 400\sin\theta\cos\theta$ A1

[6]

(b) (i) $\dfrac{\mathrm{d}P}{\mathrm{d}\theta} = 0 - 400\begin{pmatrix}(\cos\theta)(\cos\theta) \\ +(\sin\theta)(-\sin\theta)\end{pmatrix}$ (M1) for product rule

$\dfrac{\mathrm{d}P}{\mathrm{d}\theta} = -400\cos^2\theta + 400\sin^2\theta$ A1

(ii) $\dfrac{\mathrm{d}P}{\mathrm{d}\theta} = 0$

$\therefore -400\cos^2\theta + 400\sin^2\theta = 0$

By considering the graph of

$y = -400\cos^2\theta + 400\sin^2\theta$,

$\theta = 0.7853981634$. (M1) for valid approach

$\therefore \theta = 0.785 \text{ rad}$ A1

(iii) $\left.\dfrac{\mathrm{d}^2S}{\mathrm{d}\theta^2}\right|_{\theta=0.7853981634}$

$= 1600(\sin 0.7853981634)$

$\cdot(\cos 0.7853981634)$

$= 800 > 0$ R1

Thus, P attains its minimum at

$\theta = 0.785 \text{ rad}$. A1

(iv) The minimum value of P

$= 100\pi - 400\begin{pmatrix}\sin 0.7853981634 \\ \cdot\cos 0.7853981634\end{pmatrix}$ (M1) for substitution

$= 114.1592654$

$= 114$ A1

[8]

(c) $\theta = 0,\ \theta = \frac{\pi}{2}$ A2

[2]

3. (a) $Q(t)=0$ (M1) for setting equation

$t^3-12t^2+36t=0$

By considering the graph of $y=t^3-12t^2+36t$,

$t=0$ or $t=6$. A2

[3]

(b) (i) $Q'(t)=3t^2-12(2t)+36(1)$ (A1) for correct approach

$Q'(t)=3t^2-24t+36$ A1

(ii) $Q'(t)=0$

$\therefore 3t^2-24t+36=0$

$3(t-2)(t-6)=0$ (A1) for factorization

$t=2$ or $t=6$ A1

(iii) $Q''(t)=3(2t)-24(1)+0$

$Q''(t)=6t-24$ (A1) for correct approach

$Q''(2)=6(2)-24$

$Q''(2)=-12<0$ R1

Thus, Q attains its local maximum at $t=2$. A1

(iv) 32 A1

[8]

(c) (i) The minimum value of R

$=0-20$ (A1) for correct approach

$=-20$ A1

(ii) $t=0$, $t=6$ A1

[3]

4. (a) $P(t)=0$

$\therefore -t^3+9t^2-24t+720=0$ — (M1) for setting equation

By considering the graph of

$y=-t^3+9t^2-24t+720$, $t=12$. — A1

[2]

(b) (i) $P'(t)=-3t^2+9(2t)-24(1)+0$ — (A1) for correct approach

$P'(t)=-3t^2+18t-24$ — A1

(ii) $P'(t)=0$

$\therefore -3t^2+18t-24=0$

$-3(t-2)(t-4)=0$ — (A1) for factorization

$t=2$ or $t=4$ — A1

(iii) $P''(t)=-3(2t)+18(1)-0$

$P''(t)=-6t+18$ — (A1) for correct approach

$P''(2)=-6(2)+18$

$P''(2)=6>0$ — R1

Thus, P attains its local minimum at $t=2$. — A1

(iv) 700 — A1

[8]

(c) (i) 704 — A1

(ii) 720 — A1

[2]

(d) The corresponding value of t

$=0+3$ — (A1) for correct approach

$=3$ — A1

[2]

Chapter 11 Solution

Exercise 47

1. (a) $f(x)=\int \cos^3 2x \sin 2x\,\mathrm{d}x$ A1

[1]

(b) Let $u=\cos 2x$.

$\frac{\mathrm{d}u}{\mathrm{d}x}=-2\sin 2x \Rightarrow -\frac{1}{2}\mathrm{d}u=\sin 2x\mathrm{d}x$ A1

$\therefore f(x)=\int -\frac{1}{2}u^3\mathrm{d}u$ (A1) for substitution

$f(x)=-\frac{1}{8}u^4+C$

$f(x)=-\frac{1}{8}\cos^4 2x+C$ A1

[3]

(c) $3=-\frac{1}{8}\cos^4\left(2\left(\frac{\pi}{2}\right)\right)+C$ (M1) for substitution

$C=\frac{25}{8}$

$\therefore f(x)=-\frac{1}{8}\cos^4 2x+\frac{25}{8}$ A1

[2]

2. (a) $f(x)=\int 2x\sin(x^2)\mathrm{d}x$ A1

[1]

(b) Let $u=x^2$.

$\frac{\mathrm{d}u}{\mathrm{d}x}=2x \Rightarrow \mathrm{d}u=2x\mathrm{d}x$ A1

$\therefore f(x)=\int \sin u\mathrm{d}u$ (A1) for substitution

$f(x)=-\cos u+C$

$f(x)=-\cos(x^2)+C$ A1

[3]

(c) $-1=-\cos(0^2)+C$ (M1) for substitution

$C=0$

$\therefore f(x)=-\cos(x^2)$ A1

[2]

3. (a) $f(x)=\int 3x^2(x^3+1)^6\,\mathrm{d}x$ A1

[1]

(b) Let $u=x^3+1$. A1

$\frac{\mathrm{d}u}{\mathrm{d}x}=3x^2 \Rightarrow \mathrm{d}u=3x^2\mathrm{d}x$ A1

$\therefore\ f(x)=\int u^6\mathrm{d}u$ (A1) for substitution

$f(x)=\frac{1}{7}u^7+C$

$f(x)=\frac{1}{7}(x^3+1)^7+C$ A1

[4]

(c) $2=\frac{1}{7}((-1)^3+1)^7+C$ (M1) for substitution

$C=2$

$\therefore\ f(x)=\frac{1}{7}(x^3+1)^7+2$ A1

[2]

4. (a) $f(x)=\int 4x^3e^{x^4}\mathrm{d}x$ (M1) for indefinite integral

Let $u=x^4$. A1

$\frac{\mathrm{d}u}{\mathrm{d}x}=4x^3 \Rightarrow \mathrm{d}u=4x^3\mathrm{d}x$ A1

$\therefore\ f(x)=\int e^u\mathrm{d}u$ (A1) for substitution

$f(x)=e^u+C$

$f(x)=e^{x^4}+C$ A1

[5]

(b) $e^{16}-1=e^{2^4}+C$ (M1) for substitution

$C=-1$

$\therefore\ f(x)=e^{x^4}-1$ A1

[2]

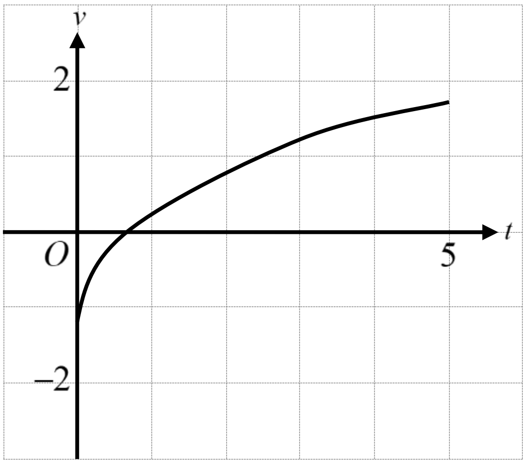
Exercise 48

1. (a) For approximately correct shape — A1

For correct x-intercept at 0.7 — A1

For approximately correct endpoints — A1

[3]



(b) The total distance travelled

$= \int_0^5 |v(t)|\,dt$ — (M1) for valid approach

$= \int_0^5 |\ln(t+0.3)|\,dt$ — (A1) for substitution

$= 4.877655148$

$= 4.88$ m — A1

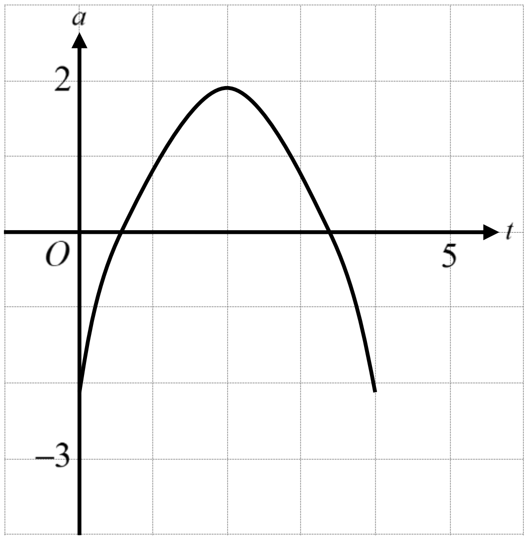
[3]

2. (a) For approximately correct shape A1

For approximately correct x-intercepts between 0 and 1 and between 3 and 4 A1

For approximately correct endpoints and maximum point A1

[3]



(b) $v(4)-v(0)=\int_0^4 a(t)\mathrm{d}t$ (A1) for correct approach

$v(4)-v(0)=\int_0^4 (-(t-2)^2+1.9)\mathrm{d}t$ (A1) for substitution

$v(4)-v(0)=2.266666667 \text{ ms}^{-1}$

$v(4)-v(0)=2.27 \text{ ms}^{-1}$ A1

[3]

3. (a) $v(t)=\int \frac{2t}{t^2+1}\,dt$ (A1) for correct approach

Let $u=t^2+1$.

$\frac{du}{dt}=2t \Rightarrow du=2t\,dt$ A1

$\therefore v(t)=\int \frac{1}{u}\,du$ (A1) for substitution

$v(t)=\ln u+C$

$v(t)=\ln(t^2+1)+C$ A1

[4]

(b) $0=\ln(0^2+1)+C$ (M1) for substitution

$C=0$

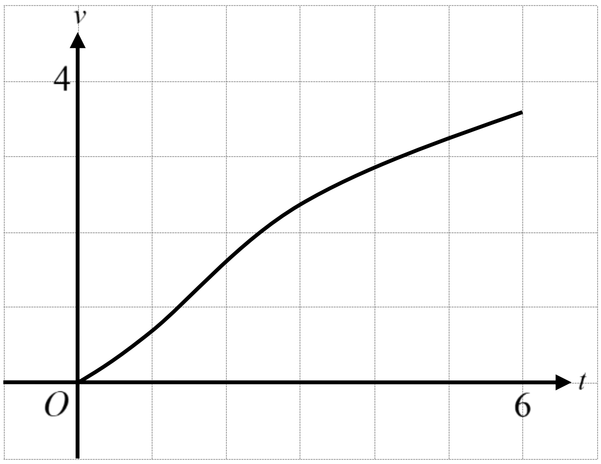
$\therefore v(t)=\ln(t^2+1)$ A1

[2]

(c) For approximately correct shape A1

For approximately correct endpoints A1

[2]



4. (a) $s(t)=\int 2t\cos(t^2)\mathrm{d}t$ (A1) for correct approach

Let $u=\sin(t^2)$.

$\frac{\mathrm{d}u}{\mathrm{d}t}=2t\cos(t^2)\Rightarrow \mathrm{d}u=2t\cos(t^2)\mathrm{d}t$ A1

$\therefore s(t)=\int \mathrm{d}u$

$s(t)=u+C$

$s(t)=\sin(t^2)+C$ A1

[3]

(b) $1=\sin(0^2)+C$ (M1) for substitution

$C=1$

$\therefore s(t)=\sin(t^2)+1$ A1

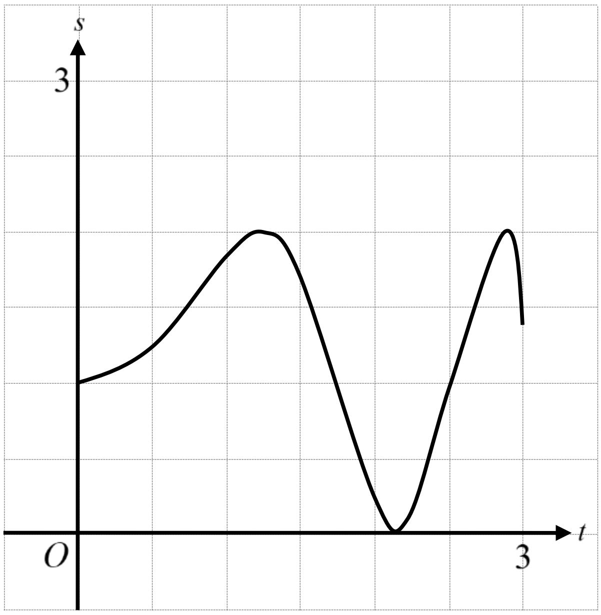
[2]

(c) For approximately correct shape A1

For approximately correct x-intercept between 2 and 3 A1

For approximately correct endpoints and maximum points A1

[3]



Exercise 49

1. (a) By considering the graph of $y=-\cos\pi x$, $x=0.5$ or $x=1.5$. (M1) for valid approach

Thus, the x-intercepts are 0.5 and 1.5. A1

[2]

(b) The area of the region

$=\int_{0.5}^{1.5}\left|-\cos\pi x\right|\mathrm{d}x$ (A1) for correct approach

$=0.6366197724$

$=0.637$ A1

[2]

2. (a) By considering the graph of $y=e^{x-1}-1$, $x=1$. (M1) for valid approach

Thus, the y-intercept is 1. A1

[2]

(b) The area of the region

$=\int_{0}^{1}\left|e^{y-1}-1\right|\mathrm{d}y$ (A1) for correct approach

$=0.3678794412$

$=0.368$ A1

[2]

3. (a) By considering the graph of $y=3e^{-x^2}-2$,

$x=-0.6367614$ or $x=0.6367614$. (M1) for valid approach

Thus, the y-intercepts are -0.637 and 0.637. A1

[2]

(b) The area of the region

$=\int_{-0.6367614}^{0.6367614}\left|3e^{-y^2}-2\right|\mathrm{d}y$ (A1) for correct approach

$=0.8143490125$

$=0.814$ A1

[2]

4. (a) By considering the graph of

$y=x^3-21x^2+138x-280$, $x=4$, $x=7$ or $x=10$. (M1) for valid approach

Thus, the y-intercepts are 4, 7 and 10. A2

[3]

(b) The total area of the region

$=\int_{4}^{10}\left|y^3-21y^2+138y-280\right|\mathrm{d}y$ (A1) for correct approach

$=40.5$ A1

[2]

Exercise 50

1. (a) The area of R

$$=\int_{\frac{2\pi}{3}}^{\frac{5\pi}{6}}\left|\frac{\sqrt{\cos\left(x-\frac{\pi}{2}\right)}}{\sin\left(x-\frac{\pi}{2}\right)}\right|dx$$ (A1) for correct approach

$=0.6475231473$

$=0.648$ A1

[2]

(b) The volume of the solid

$$=\int_{\frac{2\pi}{3}}^{\frac{5\pi}{6}}\pi\left(\frac{\sqrt{\cos\left(x-\frac{\pi}{2}\right)}}{\sin\left(x-\frac{\pi}{2}\right)}\right)^2 dx$$ (A1) for correct approach

$=2.655586579$

$=2.66$ A1

[2]

2. (a) The area of R

$=\int_0^{0.75}\left|\pi e^{\pi y}\right|dy$ (A1) for correct approach

$=9.550724074$

$=9.55$ A1

[2]

(b) The volume of the solid

$=\int_0^{0.75}\pi(\pi e^{\pi y})^2 dy$ (A1) for correct approach

$=544.3964161$

$=544$ A1

[2]

3. (a) $x=-3,\ x=3$ A2

[2]

(b) $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$

$\dfrac{y^2}{4}=1-\dfrac{x^2}{9}$

$y^2=4-\dfrac{4x^2}{9}$ A1

[1]

(c) The volume of the rugby model

$=\int_{-3}^{3}\pi\left(4-\dfrac{4x^2}{9}\right)\mathrm{d}x$ (A1) for correct approach

$=50.26548246$

$=50.3$ A1

[2]

4. (a) $2x+y-6=0$

$2x=6-y$

$x=3-\dfrac{1}{2}y$ A1

[1]

(b) The area of R

$=\int_{0}^{4}\left|3-\dfrac{1}{2}y\right|\mathrm{d}y$ (A1) for correct approach

$=8$ A1

[2]

(c) The volume of the conical frustum

$=\int_{0}^{4}\pi\left(3-\dfrac{1}{2}y\right)^2\mathrm{d}y$ (A1) for correct approach

$=54.45427266$

$=54.5$ A1

[2]

Exercise 51

1.	(a)	(i)	0	A1	
		(ii)	e^6+2	A1	
					[2]
	(b)	3		A1	
					[1]
	(c)	The area of R			
		$=\int_0^3 (e^{2x}-1)\mathrm{d}x+\int_3^{e^6+2}(e^6+2-x)\mathrm{d}x$		(A2) for correct approach	
		$=81172.68131$			
		$=81200$		A1	
					[3]
2.	(a)	(i)	1	A1	
		(ii)	$2e^2$	A1	
					[2]
	(b)	$x=7.3890561$			
		$x=7.39$		A1	
					[1]
	(c)	The volume of the solid			
		$=\int_1^{7.3890561}\pi(2\ln x)^2\mathrm{d}x+\int_{7.3890561}^{2e^2}\pi\left(8-\frac{4}{e^2}x\right)^2\mathrm{d}x$		(A2) for correct approach	
		$=284.3793169$			
		$=284$		A1	
					[3]
3.	(a)	(i)	1	A1	
		(ii)	$y=-0.950212931$		
			$y=-0.950$	A1	
					[2]
	(b)	0		A1	
					[1]
	(c)	The area of R			
		$=\int_{-0.950212931}^{0}\left|-3+\ln\left|\frac{1}{1+y}\right|\right|\mathrm{d}y+\int_0^1\lvert 3y-3\rvert\mathrm{d}y$		(A2) for correct approach	
		$=3.549787068$			
		$=3.55$		A1	
					[3]

4. (a) (i) 1 A1

(ii) 19 A1

[2]

(b) $y = \frac{1}{2}x + 1$

$2y = x + 2$

$x = 2y - 2$ A1

[1]

(c) 3 A1

[1]

(d) The volume of the solid

$= \int_1^3 \pi(2y-2)^2 \, dy + \int_3^{19} \pi(4 - \sqrt{y-3})^2 \, dy$ (A2) for correct approach

$= 167.551608$

$= 168$ A1

[3]

Exercise 52

1.	(a)	(i)	$f'(a)\times -4=-1$	(M1) for valid approach
			$f'(a)=\frac{1}{4}$	A1
		(ii)	$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$	(A1) for correct approach
			$f'(a)=\frac{1}{2\sqrt{a}}$	
			$\therefore \frac{1}{2\sqrt{a}}=\frac{1}{4}$	(M1) for setting equation
			$\sqrt{a}=2$	
			$a=4$	A1
		(iii)	$\frac{\sqrt{4}-0}{4-b}=-4$	(M1) for setting equation
			$\frac{2}{4-b}=-4$	
			$-\frac{1}{2}=4-b$	
			$b=\frac{9}{2}$	A1
		(iv)	The equation of [PQ]:	
			$y=-4x+c$	
			$0=-4\left(\frac{9}{2}\right)+c$	(M1) for substitution
			$c=18$	
			$\therefore y=-4x+18$	A1

[9]

	(b)	The required area	
		$=\int_0^4 f(x)\mathrm{d}x+\frac{(4.5-4)(2-0)}{2}$	M1A1
		$=\int_0^4 x^{\frac{1}{2}}\mathrm{d}x+\frac{(0.5)(2)}{2}$	(A1) for correct approach
		$=\frac{35}{6}$	A1

[4]

2. (a) (i) $f'(x) = 2x$ (A1) for correct approach

$\therefore f'(a) = 2a$ A1

$f(a) = a^2$

The gradient of [PQ]

$= f'(a)$

$= \dfrac{f(a) - 0}{a - 1}$ (A1) for substitution

$= \dfrac{a^2}{a - 1}$ A1

(ii) $2a = \dfrac{a^2}{a - 1}$ (M1) for setting equation

$2 = \dfrac{a}{a - 1}$

$2a - 2 = a$

$a = 2$ A1

(iii) The equation of [PQ]:

$y = 4x + c$

$0 = 4(1) + c$ (M1) for substitution

$c = -4$

$\therefore y = 4x - 4$ A1

[8]

(b) The required area

$= \int_0^2 f(x)\mathrm{d}x - \dfrac{(2-1)(4-0)}{2}$ M1A1

$= \int_0^2 x^2\mathrm{d}x - \dfrac{(1)(4)}{2}$ (A1) for correct approach

$= \dfrac{2}{3}$ A1

[4]

3. (a) $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$ (A1) for correct approach

$f'(h)=\frac{1}{2\sqrt{h}}$ A1

[2]

(b) (i) $\frac{1}{2\sqrt{h}}=\frac{1}{6}$ (M1) for setting equation

$2\sqrt{h}=6$

$\sqrt{h}=3$

$h=9$ A1

(ii) The equation of $[\mathrm{AB}]$:

$y=\frac{1}{6}x+c$

$3=\frac{1}{6}(9)+c$ (M1) for substitution

$c=\frac{3}{2}$

$\therefore y=\frac{1}{6}x+\frac{3}{2}$ A1

(iii) $0=\frac{1}{6}b+\frac{3}{2}$ (M1) for setting equation

$\frac{1}{6}b=-\frac{3}{2}$

$b=-9$ A1

[6]

(c) The required area

$=\frac{(9-(-9))(3-0)}{2}-\int_0^9 f(x)\mathrm{d}x$ M1A1

$=\frac{(18)(3)}{2}-\int_0^9 x^{\frac{1}{2}}\mathrm{d}x$ (A1) for correct approach

$=9$ A1

[4]

4. (a) $f'(x)=3x^2$ — (A1) for correct approach

$f'(h)=3h^2$ — A1

[2]

(b) (i) $3h^2=\frac{3}{4}$ — (M1) for setting equation

$h^2=\frac{1}{4}$

$h=-\frac{1}{2}$ or $h=\frac{1}{2}$ (*Rejected*) — A1

(ii) $-\frac{4}{3}$ — A1

(iii) The equation of [AB]:

$y=-\frac{4}{3}x+c$

$\left(-\frac{1}{2}\right)^3=-\frac{4}{3}\left(-\frac{1}{2}\right)+c$ — (M1) for substitution

$c=-\frac{19}{24}$

$\therefore y=-\frac{4}{3}x-\frac{19}{24}$ — A1

(iv) $0=-\frac{4}{3}b-\frac{19}{24}$ — (M1) for setting equation

$\frac{4}{3}b=-\frac{19}{24}$

$b=-\frac{19}{32}$ — A1

[7]

(c) The required area

$=\frac{\left(-\frac{1}{2}-\left(-\frac{19}{32}\right)\right)\left(0-\left(-\frac{1}{8}\right)\right)}{2}+\int_{-\frac{1}{2}}^{0}|f(x)|\,\mathrm{d}x$ — M1A1

$=\frac{\left(\frac{3}{32}\right)\left(\frac{1}{8}\right)}{2}+\int_{-\frac{1}{2}}^{0}|x^3|\,\mathrm{d}x$ — (A1) for correct approach

$=0.021484375$

$=0.0215$ — A1

[4]

Exercise 53

1. (a) The initial velocity

$= v(0)$

$= -(0-4)^3$ (M1) for substitution

$= 64 \text{ ms}^{-1}$ A1

[2]

(b) $v(t) = -27$ (M1) for setting equation

$-(t-4)^3 = -27$

$t - 4 = 3$ (A1) for correct approach

$t = 7$ A1

[3]

(c) The total distance travelled

$= \int_0^7 |v(t)| \, dt$ (M1) for valid approach

$= \int_0^7 |-(t-4)^3| \, dt$ (A1) for substitution

$= 84.24999949 \text{ m}$

$= 84.2 \text{ m}$ A1

[3]

(d) $a(t) = v'(t)$

$a(t) = -3(t-4)^2(1)$ (A1) for correct approach

$a(t) = -3(t-4)^2$ A1

[2]

(e) $v(t) < 0$ and $a(t) < 0$

By considering the graph of $y = -(t-4)^3$ and

$y = -3(t-4)^2$, $t > 4$ and $t \neq 4$. R2

$\therefore 4 < t \leq 8$ A1

[3]

(f) $s(t) = \int v(t) dt$

$s(t) = \int -(t-4)^3 dt$ (A1) for correct approach

$s(t) = -\frac{1}{4}(t-4)^4 + C$ A1

$28 = -\frac{1}{4}(0-4)^4 + C$ (M1) for substitution

$C = 92$

$\therefore s(t) = -\frac{1}{4}(t-4)^4 + 92$ A1

[4]

2. (a) $s(t)=\int v(t)\mathrm{d}t$

$s(t)=\int(-2t^3+12t^2-24t+16)\mathrm{d}t$ (A1) for correct approach

$s(t)=-2\left(\frac{1}{4}t^4\right)+12\left(\frac{1}{3}t^3\right)-24\left(\frac{1}{2}t^2\right)+16t+C$

$s(t)=-\frac{1}{2}t^4+4t^3-12t^2+16t+C$ A1

$0=-\frac{1}{2}(0)^4+4(0)^3-12(0)^2+16(0)+C$ (M1) for substitution

$C=0$

$\therefore s(t)=-\frac{1}{2}t^4+4t^3-12t^2+16t$ A1

[4]

(b) The displacement

$=s(3.3)$

$=-\frac{1}{2}(3.3)^4+4(3.3)^3-12(3.3)^2+16(3.3)$ (M1) for substitution

$=6.57195\text{ m}$ A1

[2]

(c) The total distance travelled

$=\int_0^{3.3}|v(t)|\mathrm{d}t$ (M1) for valid approach

$=\int_0^{3.3}\left|-2t^3+12t^2-24t+16\right|\mathrm{d}t$ (A1) for substitution

$=9.428049981\text{ m}$

$=9.43\text{ m}$ A1

[3]

(d) $a(t)=v'(t)$

$a(t)=-2(3t^2)+12(2t)-24(1)+0$ (A1) for correct approach

$a(t)=-6t^2+24t-24$ A1

[2]

(e) $v(t)>0$ and $a(t)<0$

By considering the graph of

$y=-2t^3+12t^2-24t+16$ and $y=-6t^2+24t-24$,

$t<2$ and $t\neq 2$. R2

$\therefore 0\le t<2$ A1

[3]

3. (a) (i) $s(t)=\int v(t)\mathrm{d}t$

$s(t)=\int \pi\cos\pi t\mathrm{d}t$ A1

Let $u=\pi t$. A1

$\frac{\mathrm{d}u}{\mathrm{d}t}=\pi \Rightarrow \mathrm{d}u=\pi\mathrm{d}t$ A1

$\therefore s(t)=\int \cos u\mathrm{d}u$ AG

(ii) $s(t)=\int \cos u\mathrm{d}u$

$s(t)=\sin u+C$

$s(t)=\sin\pi t+C$ A1

$1=\sin 0+C$ (M1) for substitution

$C=1$

$\therefore s(t)=\sin\pi t+1$ A1

[6]

(b) $s(t)=0$

$\sin\pi t+1=0$ (M1) for setting equation

By considering the graph of $y=\sin\pi t+1$, $t=\frac{3}{2}$ or

$t=\frac{7}{2}$. A2

[3]

(c) (i) $a(t)=v'(t)$

$a(t)=\pi(-\sin\pi t)(\pi)$ (M1) for chain rule

$a(t)=-\pi^2\sin\pi t$ A1

(ii) $a(t)>0$

$-\pi^2\sin\pi t>0$ (M1) for setting inequality

By considering the graph of $y=-\pi^2\sin\pi t$,

$1<t<2$ or $3<t<4$. A2

[5]

(d) 5 A1

[1]

4. (a) $v(t)=\int a(t)\mathrm{d}t$

$v(t)=\int(4t^3-33t^2+88t-76)\mathrm{d}t$ (A1) for correct approach

$v(t)=4\left(\frac{1}{4}t^4\right)-33\left(\frac{1}{3}t^3\right)+88\left(\frac{1}{2}t^2\right)-76t+C$

$v(t)=t^4-11t^3+44t^2-76t+C$ A1

$48=0^4-11(0)^3+44(0)^2-76(0)+C$ (M1) for substitution

$C=48$

$\therefore v(t)=t^4-11t^3+44t^2-76t+48$ A1

[4]

(b) $s(t)=\int v(t)\mathrm{d}t$

$s(t)=\int(t^4-11t^3+44t^2-76t+48)\mathrm{d}t$ (A1) for correct approach

$s(t)=\frac{1}{5}t^5-11\left(\frac{1}{4}t^4\right)+44\left(\frac{1}{3}t^3\right)$

$-76\left(\frac{1}{2}t^2\right)+48t+D$

$s(t)=\frac{1}{5}t^5-\frac{11}{4}t^4+\frac{44}{3}t^3-38t^2+48t+D$ A1

$0=\frac{1}{5}(0)^5-\frac{11}{4}(0)^4+\frac{44}{3}(0)^3-38(0)^2+48(0)+D$ (M1) for substitution

$D=0$

$\therefore s(t)=\frac{1}{5}t^5-\frac{11}{4}t^4+\frac{44}{3}t^3-38t^2+48t$ A1

[4]

(c) $s(t)>0$ and $a(t)<0$

$0<t\le 5$ and $(t<2$ or $2.6096118<t<3.6403882)$ R2

$\therefore 0<t<2$ or $2.6096118<t<3.6403882$

$0<t<2$ or $2.61<t<3.64$ A2

[4]

(d) (i) 3 A1

(ii) 2 A1

[2]

Chapter 12 Solution

Exercise 54

1. (a) $\frac{dx}{dt} = \pi^2 x \sin \pi t$

$\frac{1}{x} dx = \pi^2 \sin \pi t dt$ (M1) for valid approach

$\therefore \int \frac{1}{x} dx = \int \pi^2 \sin \pi t dt$ A1

[2]

(b) Let $u = \pi t$.

$\frac{du}{dt} = \pi \Rightarrow du = \pi dt$ A1

$\therefore \int \frac{1}{x} dx = \int \pi \sin u du$ (A1) for correct working

$\ln x = -\pi \cos u + C$

$\ln x = -\pi \cos \pi t + C$ A1

[3]

(c) $\ln 1 = -\pi \cos \pi(0.5) + C$ (M1) for substitution

$0 = 0 + C$

$C = 0$ (A1) for correct value

$\therefore \ln x = -\pi \cos \pi t$

$x = e^{-\pi \cos \pi t}$ A1

[3]

2. (a) $\frac{dy}{dx} = \frac{y}{2x+3}$

$\frac{1}{y}dy = \frac{1}{2x+3}dx$ (M1) for valid approach

$\therefore \int \frac{1}{y}dy = \int \frac{1}{2x+3}dx$ A1

[2]

(b) Let $u = 2x+3$.

$\frac{du}{dx} = 2 \Rightarrow \frac{1}{2}du = dx$ A1

$\therefore \int \frac{1}{y}dy = \int \frac{1}{2u}du$ (A1) for correct working

$\ln y = \frac{1}{2}\ln u + C$

$\ln y = \frac{1}{2}\ln(2x+3) + C$ A1

[3]

(c) $\ln 12 = \frac{1}{2}\ln(2(3)+3) + C$ (M1) for substitution

$\ln 12 = \frac{1}{2}\ln 9 + C$

$\ln 12 = \ln 3 + C$

$C = \ln 4$ (A1) for correct value

$\therefore \ln y = \frac{1}{2}\ln(2x+3) + \ln 4$

$y = e^{\frac{1}{2}\ln(2x+3)+\ln 4}$ A1

[3]

3. (a) $\dfrac{dy}{dx} = \dfrac{2}{y^2}$

$y^2 dy = 2dx$ (M1) for valid approach

$\int y^2 dy = \int 2dx$ (A1) for correct approach

$\dfrac{1}{3}y^3 = 2x + C$ A1

[3]

(b) $\dfrac{1}{3}(9)^3 = 2(121) + C$ (M1) for substitution

$243 = 242 + C$

$C = 1$ (A1) for correct value

$\therefore \dfrac{1}{3}y^3 = 2x + 1$

$y^3 = 6x + 3$

$y = \sqrt[3]{6x+3}$ A1

[3]

(c) $y = \sqrt[3]{6\left(\dfrac{61}{3}\right) + 3}$ (M1) for substitution

$y = \sqrt[3]{125}$

$y = 5$ A1

[2]

4. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = -y^3 e^x$

$\frac{1}{y^3}\,\mathrm{d}y = -e^x \mathrm{d}x$ (M1) for valid approach

$\int \frac{1}{y^3}\,\mathrm{d}y = \int -e^x \mathrm{d}x$ (A1) for correct approach

$-\frac{1}{2y^2} = -e^x + C$

$\therefore \frac{1}{2y^2} = e^x + C$ A1

[3]

(b) $\frac{1}{2(0.25)^2} = e^{\ln 2} + C$ (M1) for substitution

$8 = 2 + C$

$C = 6$ (A1) for correct value

$\therefore \frac{1}{2y^2} = e^x + 6$

$y^2 = \frac{1}{2e^x + 12}$

$y = \frac{1}{\sqrt{2e^x + 12}}$ A1

[3]

(c) $y = \frac{1}{\sqrt{2e^{\ln 12} + 12}}$ (M1) for substitution

$y = \frac{1}{\sqrt{36}}$

$y = \frac{1}{6}$ A1

[2]

Exercise 55

1. (a) $\det(\mathbf{M}-\lambda\mathbf{I})=\begin{vmatrix}5-\lambda & 6\\ 4 & 3-\lambda\end{vmatrix}$ (M1) for valid approach

$\det(\mathbf{M}-\lambda\mathbf{I})=(5-\lambda)(3-\lambda)-(6)(4)$

$\det(\mathbf{M}-\lambda\mathbf{I})=15-5\lambda-3\lambda+\lambda^2-24$

$\det(\mathbf{M}-\lambda\mathbf{I})=\lambda^2-8\lambda-9$ A1

[2]

(b) $\lambda_1=-1,\ \lambda_2=9$ A2

[2]

(c) $\mathbf{v}_1=\begin{pmatrix}-1\\ 1\end{pmatrix},\ \mathbf{v}_2=\begin{pmatrix}\frac{3}{2}\\ 1\end{pmatrix}$ A2

[2]

(d) $\mathbf{X}=Ae^{-t}\begin{pmatrix}-1\\ 1\end{pmatrix}+Be^{9t}\begin{pmatrix}\frac{3}{2}\\ 1\end{pmatrix}$ A2

[2]

2. (a) $\det(\mathbf{M}-\lambda\mathbf{I})=\begin{vmatrix}2-\lambda & 15\\ 2 & 1-\lambda\end{vmatrix}$ (M1) for valid approach

$\det(\mathbf{M}-\lambda\mathbf{I})=(2-\lambda)(1-\lambda)-(15)(2)$

$\det(\mathbf{M}-\lambda\mathbf{I})=2-2\lambda-\lambda+\lambda^2-30$

$\det(\mathbf{M}-\lambda\mathbf{I})=\lambda^2-3\lambda-28$ A1

[2]

(b) $\lambda_1=-4,\ \lambda_2=7$ A2

[2]

(c) $\mathbf{v}_1=\begin{pmatrix}-\frac{5}{2}\\ 1\end{pmatrix},\ \mathbf{v}_2=\begin{pmatrix}3\\ 1\end{pmatrix}$ A2

[2]

(d) (i) $x=-\frac{5}{2}Ae^{-4t}+3Be^{7t}$ A1

(ii) $y=Ae^{-4t}+Be^{7t}$ A1

[2]

3. (a) $\det(\mathbf{M}-\lambda\mathbf{I})=\begin{vmatrix}4-\lambda & 2\\ -9 & -2-\lambda\end{vmatrix}$ (M1) for valid approach

$\det(\mathbf{M}-\lambda\mathbf{I})=(4-\lambda)(-2-\lambda)-(2)(-9)$

$\det(\mathbf{M}-\lambda\mathbf{I})=-8-4\lambda+2\lambda+\lambda^2+18$

$\det(\mathbf{M}-\lambda\mathbf{I})=\lambda^2-2\lambda+10$ A1

[2]

(b) $1+3\text{i}$, $1-3\text{i}$ A2

[2]

(c) $\mathbf{X}=e^{t}(Ae^{3t\text{i}}\mathbf{v}_1+Be^{-3t\text{i}}\mathbf{v}_2)$ A2

[2]

(d) The phase portrait is an unstable spiral. A1

[1]

4. (a) $\det(\mathbf{M}-\lambda\mathbf{I})=\begin{vmatrix}-8-\lambda & -1\\ 13 & -2-\lambda\end{vmatrix}$ (M1) for valid approach

$\det(\mathbf{M}-\lambda\mathbf{I})=(-8-\lambda)(-2-\lambda)-(-1)(13)$

$\det(\mathbf{M}-\lambda\mathbf{I})=16+8\lambda+2\lambda+\lambda^2+13$

$\det(\mathbf{M}-\lambda\mathbf{I})=\lambda^2+10\lambda+29$ A1

[2]

(b) $-5+2\text{i}$, $-5-2\text{i}$ A2

[2]

(c) $\mathbf{X}=e^{-5t}(Ae^{2t\text{i}}\mathbf{v}_1+Be^{-2t\text{i}}\mathbf{v}_2)$ A2

[2]

(d) The phase portrait is a stable spiral. A1

[1]

Exercise 56

1. (a) $\begin{cases} x_{n+1} = x_n + 0.1 \\ y_{n+1} = y_n + 0.1 \left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_{(x_n,\, y_n)} \end{cases}$ (M1) for valid approach

$x_0 = 4,\ y_0 = 10$ (A1) for correct values

$x_1 = 4 + 0.1 = 4.1$

$y_1 = 10 + 0.1\left(\dfrac{(4)(10)}{4^2 - 9}\right) = 10.57142857$

$y_1 = 10.6$ A1

[3]

(b) (i) $y_2 = 11.12639473$

$y_2 = 11.12639$ A1

(ii) $y_3 = 11.66726114$

$y_3 = 11.7$ A1

[2]

2. (a) $\begin{cases} x_{n+1} = x_n + 0.2 \\ y_{n+1} = y_n + 0.2 \left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_{(x_n,\, y_n)} \end{cases}$ (M1) for valid approach

$x_0 = 0,\ y_0 = 2$ (A1) for correct values

$x_1 = 0 + 0.2 = 0.2$

$y_1 = 2 + 0.2(e^0 + 4(2)) = 3.8$ A1

[3]

(b) (i) $y_2 = 7.084280552$

$y_2 = 7.084281$ A1

(ii) $y_3 = 13.05006993$

$y_3 = 13.050070$ A1

(iii) $y_4 = 23.85454964$

$y_4 = 23.9$ A1

[3]

3. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x} + \frac{1}{x^3}$

$$\begin{cases} x_{n+1} = x_n + 0.25 \\ y_{n+1} = y_n + 0.25 \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(x_n,\, y_n)} \end{cases}$$ (M1) for valid approach

$x_0 = 1,\ y_0 = 2$ (A1) for correct values

$x_1 = 1 + 0.25 = 1.25$

$y_1 = 2 + 0.25\left(\frac{2}{1} + \frac{1}{1^3}\right) = 2.75$ A1

[3]

(b) (i) $y_2 = 3.428$ A1

(ii) $y_3 = 4.073407407$

$y_3 = 4.0734$ A1

(iii) $y_4 = 4.701969982$

$y_4 = 4.70$ A1

[3]

4. (a) $\frac{dy}{dx} + x - xy = 0$

$\frac{dy}{dx} = xy - x$

$\begin{cases} x_{n+1} = x_n + 0.2 \\ y_{n+1} = y_n + 0.2 \left.\frac{dy}{dx}\right|_{(x_n, y_n)} \end{cases}$ (M1) for valid approach

$x_0 = 3, y_0 = 3$ (A1) for correct values

$x_1 = 3 + 0.2 = 3.2$

$y_1 = 3 + 0.2((3)(3) - 3) = 4.2$ A1

[3]

(b) (i) $y_2 = 6.248$ A1

(ii) $y_3 = 9.81664$

$y_3 = 9.8166$ A1

(iii) $y_4 = 16.1646208$

$y_4 = 16.1646$ A1

(iv) $y_5 = 27.68973261$

$y_5 = 27.7$ A1

[4]

Exercise 57

1. (a) (i)

$$\begin{cases} x_{n+1} = x_n + 0.15\left.\dfrac{\mathrm{d}x}{\mathrm{d}t}\right|_{(t_n, x_n, y_n)} \\ y_{n+1} = y_n + 0.15\left.\dfrac{\mathrm{d}y}{\mathrm{d}t}\right|_{(t_n, x_n, y_n)} \\ t_{n+1} = t_n + 0.15 \end{cases}$$

(M1) for valid approach

$t_0 = 1$, $x_0 = 5$, $y_0 = 2$ (A1) for correct values

$t_1 = 1 + 0.15 = 1.15$

$x_1 = 5 + 0.15(5 + (2)(1)) = 6.05$ A1

(ii) $y_1 = 2 + 0.15((5)(1) + 2) = 3.05$ A1

[4]

(b) (i) $x_2 = 7.483625$

$x_2 = 7.48$ A1

(ii) $y_2 = 4.551125$

$y_2 = 4.55$ A1

[2]

2. (a) (i)

$$\begin{cases} x_{n+1} = x_n + 0.01\left.\dfrac{\mathrm{d}x}{\mathrm{d}t}\right|_{(t_n,\, x_n,\, y_n)} \\ y_{n+1} = y_n + 0.01\left.\dfrac{\mathrm{d}y}{\mathrm{d}t}\right|_{(t_n,\, x_n,\, y_n)} \\ t_{n+1} = t_n + 0.01 \end{cases}$$

(M1) for valid approach

$t_0 = 0.97$, $x_0 = 1.1$, $y_0 = 1$ (A1) for correct values

$t_1 = 0.97 + 0.01 = 0.98$

$x_1 = 1.1 + 0.01\big((2-1)(0.97)\big) = 1.1097$

$x_1 = 1.11$ A1

(ii) $y_1 = 1 + 0.01\big((2-1.1)(0.97)\big) = 1.00873$

$y_1 = 1.01$ A1

[4]

(b) (i) $x_2 = 1.119414446$

$x_2 = 1.12$ A1

(ii) $y_2 = 1.01745494$

$y_2 = 1.02$ A1

(iii) $x_3 = 1.129141642$

$x_3 = 1.13$ A1

(iv) $y_3 = 1.026172737$

$y_3 = 1.03$ A1

[4]

3. (a) $\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = -8u - 15x \\ \dfrac{\mathrm{d}x}{\mathrm{d}t} = u \end{cases}$ A1

[1]

(b) (i) $\begin{cases} u_{n+1} = u_n + 0.1\left.\dfrac{\mathrm{d}u}{\mathrm{d}t}\right|_{(t_n,\, u_n,\, x_n)} \\ x_{n+1} = x_n + 0.1\left.\dfrac{\mathrm{d}x}{\mathrm{d}t}\right|_{(t_n,\, u_n,\, x_n)} \\ t_{n+1} = t_n + 0.1 \end{cases}$ (M1) for valid approach

$t_0 = 2$, $u_0 = 1$, $x_0 = 0$ (A1) for correct values

$t_1 = 2 + 0.1 = 2.1$

$u_1 = 1 + 0.1\big(-8(1) - 15(0)\big) = 0.2$ A1

(ii) $x_1 = 0 + 0.1(1) = 0.1$ A1

[4]

(c) $x_2 = 0.12$ A1

[1]

4. (a) $\begin{cases} \dfrac{du}{dt} = 12u - 36x \\ \dfrac{dx}{dt} = u \end{cases}$ A1

[1]

(b) (i) $\begin{cases} u_{n+1} = u_n + 0.05 \left.\dfrac{du}{dt}\right|_{(t_n, u_n, x_n)} \\ x_{n+1} = x_n + 0.05 \left.\dfrac{dx}{dt}\right|_{(t_n, u_n, x_n)} \\ t_{n+1} = t_n + 0.05 \end{cases}$ (M1) for valid approach

$t_0 = 0$, $u_0 = 3$, $x_0 = 2$ (A1) for correct values

$t_1 = 0 + 0.05 = 0.05$

$u_1 = 3 + 0.05\big(12(3) - 36(2)\big) = 1.2$ A1

(ii) $x_1 = 2 + 0.05(3) = 2.15$ A1

[4]

(c) $x_2 = 2.21$ A1

[1]

Exercise 58

1. (a) $(0, 0)$, $(6, 1.5)$ A2

[2]

(b) (i) $\frac{dy}{dx} = \frac{(x-6)y}{(3-2y)x}$ A1

(ii) $\left.\frac{dy}{dx}\right|_{t=0} = \frac{(2-6)(2.5)}{(3-2(2.5))(2)}$ (M1) for substitution

$\left.\frac{dy}{dx}\right|_{t=0} = 2.5$ A1

(iii) The population of shark is decreasing at the beginning. A1

[4]

2. (a) $(0, 0)$, $(8, 0.4)$ A2

[2]

(b) (i) $\frac{dy}{dx} = \frac{(x-8)y}{(2-5y)x}$ A1

(ii) $\left.\frac{dy}{dx}\right|_{t=0} = \frac{(4-8)(3)}{(2-5(3))(4)}$ (M1) for substitution

$\left.\frac{dy}{dx}\right|_{t=0} = \frac{3}{13}$ A1

(iii) $0 \le y < 0.4$ A1

[4]

3. (a) The population of crocodile is increasing at the beginning. A1

[1]

(b) (i)

$$\begin{cases} x_{n+1} = x_n + 0.2 \left.\dfrac{\mathrm{d}x}{\mathrm{d}t}\right|_{(t_n,\, x_n,\, y_n)} \\ y_{n+1} = y_n + 0.2 \left.\dfrac{\mathrm{d}y}{\mathrm{d}t}\right|_{(t_n,\, x_n,\, y_n)} \\ t_{n+1} = t_n + 0.2 \end{cases}$$

(M1) for valid approach

$t_0 = 0,\ x_0 = 0.5,\ y_0 = 0.5$ (A1) for correct values

$t_1 = 0 + 0.2 = 0.2$

$x_1 = 0.5 + 0.2\big((5 - 8(0.5))(0.5)\big) = 0.6$

Thus, the approximate value of the population of leopard is 600. A1

(ii) $y_1 = 0.5 + 0.2\big((7(0.5) - 3)(0.5)\big) = 0.55$

Thus, the approximate value of the population of crocodile is 550. A1

[4]

4. (a) $x > \dfrac{10}{3}$ A1

[1]

(b) (i)

$$\begin{cases} x_{n+1} = x_n + 0.1\left.\dfrac{\mathrm{d}x}{\mathrm{d}t}\right|_{(t_n,\,x_n,\,y_n)} \\ y_{n+1} = y_n + 0.1\left.\dfrac{\mathrm{d}y}{\mathrm{d}t}\right|_{(t_n,\,x_n,\,y_n)} \\ t_{n+1} = t_n + 0.1 \end{cases}$$

(M1) for valid approach

$t_0 = 0,\ x_0 = 3,\ y_0 = 3$ (A1) for correct values

$t_1 = 0 + 0.1 = 0.1$

$x_1 = 3 + 0.1\big((12 - 5(3))(3)\big) = 2.1$

Thus, the approximate numbers of soldiers from country X is 210. A1

(ii) $y_1 = 3 + 0.1\big((3(3) - 10)(3)\big) = 2.7$

Thus, the approximate numbers of soldiers from country Y is 270. A1

[4]

Exercise 59

1. (a) (0, 0) A1

[1]

(b) $\det(\mathbf{M}-\lambda\mathbf{I})=\begin{vmatrix}-1-\lambda & -5\\ 1 & -7-\lambda\end{vmatrix}$ (M1) for valid approach

$\det(\mathbf{M}-\lambda\mathbf{I})=(-1-\lambda)(-7-\lambda)-(-5)(1)$

$\det(\mathbf{M}-\lambda\mathbf{I})=7+\lambda+7\lambda+\lambda^2+5$

$\det(\mathbf{M}-\lambda\mathbf{I})=\lambda^2+8\lambda+12$ A1

[2]

(c) $\lambda_1=-6$, $\lambda_2=-2$ A2

[2]

(d) $\mathbf{v}_1=\begin{pmatrix}1\\1\end{pmatrix}$, $\mathbf{v}_2=\begin{pmatrix}5\\1\end{pmatrix}$ A2

[2]

(e) (i) $\mathbf{X}=Ae^{-6t}\begin{pmatrix}1\\1\end{pmatrix}+Be^{-2t}\begin{pmatrix}5\\1\end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix}9\\1\end{pmatrix}=Ae^{-6(0)}\begin{pmatrix}1\\1\end{pmatrix}+Be^{-2(0)}\begin{pmatrix}5\\1\end{pmatrix}$ (M1) for substitution

$\begin{cases}9=A+5B\\1=A+B\end{cases}$

By solving this system, $A=-1$ and $B=2$. (A1) for correct values

$\therefore x=-e^{-6t}+10e^{-2t}$ A1

(ii) $y=-e^{-6t}+2e^{-2t}$ A1

[5]

(f) The required coordinates

$=(-e^{-6(1)}+10e^{-2(1)}, -e^{-6(1)}+2e^{-2(1)})$ (M1) for substitution

$=(1.35087408, 0.268191814)$

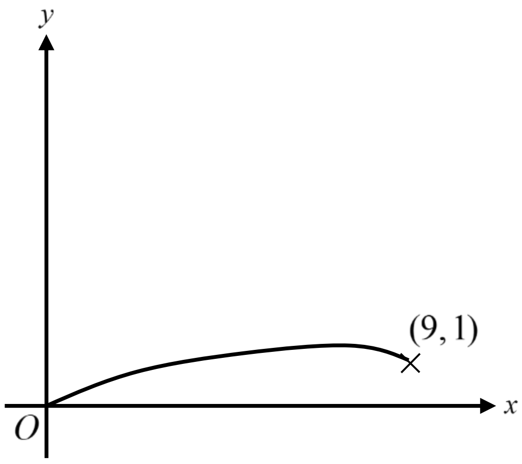
$=(1.35, 0.268)$ A1

[2]

(g) For starting at $(9,1)$ A1

For approaching the origin A1

[2]



2. (a) $y > 3$ A1

[1]

(b) $\det(\mathbf{M}-\lambda\mathbf{I}) = \begin{vmatrix} -3-\lambda & 0 \\ -1 & 4-\lambda \end{vmatrix}$ (M1) for valid approach

$\det(\mathbf{M}-\lambda\mathbf{I}) = (-3-\lambda)(4-\lambda)-(0)(-1)$

$\det(\mathbf{M}-\lambda\mathbf{I}) = -12+3\lambda-4\lambda+\lambda^2$

$\det(\mathbf{M}-\lambda\mathbf{I}) = \lambda^2-\lambda-12$ A1

[2]

(c) $\lambda_1 = -3,\ \lambda_2 = 4$ A2

[2]

(d) $\mathbf{v}_1 = \begin{pmatrix} 7 \\ 1 \end{pmatrix},\ \mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ A2

[2]

(e) (i) $\mathbf{X} = Ae^{-3t}\begin{pmatrix} 7 \\ 1 \end{pmatrix} + Be^{4t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix} 7 \\ 2 \end{pmatrix} = Ae^{-3(0)}\begin{pmatrix} 7 \\ 1 \end{pmatrix} + Be^{4(0)}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (M1) for substitution

$\begin{cases} 7 = 7A \\ 2 = A+B \end{cases}$

By solving this system, $A=1$ and $B=1$. (A1) for correct values

$\therefore x = 7e^{-3t}$ A1

(ii) $y = e^{-3t}+e^{4t}$ A1

[5]

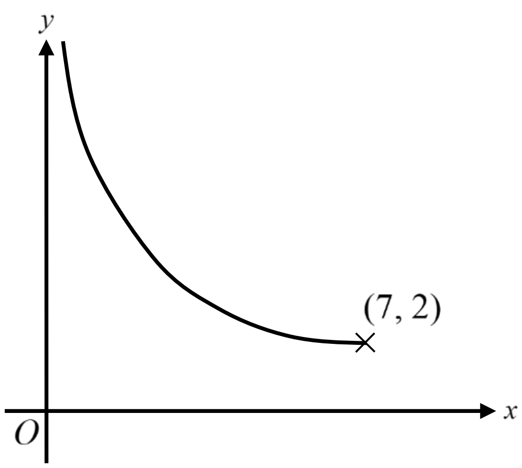
(f) The population of horse will approach 0. A1

[1]

(g) For starting at $(7, 2)$ A1

For correct asymptotic behaviour A1

[2]



3. (a) $\begin{cases} \dfrac{dv}{dt} = 3v + 4x \\ \dfrac{dx}{dt} = v \end{cases}$ A1

[1]

(b) (i) $\begin{cases} v_{n+1} = v_n + 0.1\left.\dfrac{dv}{dt}\right|_{(t_n,\, v_n,\, x_n)} \\ x_{n+1} = x_n + 0.1\left.\dfrac{dx}{dt}\right|_{(t_n,\, v_n,\, x_n)} \\ t_{n+1} = t_n + 0.1 \end{cases}$ (M1) for valid approach

$t_0 = 0,\ v_0 = 1,\ x_0 = 0$ (A1) for correct values

$t_1 = 0 + 0.1 = 0.1$

$v_1 = 1 + 0.1\big(3(1) + 4(0)\big) = 1.3$ A1

(ii) $x_1 = 0 + 0.1(1) = 0.1$ A1

[4]

(c) (i) 0.23 m A1

(ii) 0.403 m A1

[2]

(d) $\det(\mathbf{M} - \lambda\mathbf{I}) = \begin{vmatrix} 3-\lambda & 4 \\ 1 & 0-\lambda \end{vmatrix}$ (M1) for valid approach

$\det(\mathbf{M} - \lambda\mathbf{I}) = (3-\lambda)(-\lambda) - (4)(1)$

$\det(\mathbf{M} - \lambda\mathbf{I}) = -3\lambda + \lambda^2 - 4$

$\det(\mathbf{M} - \lambda\mathbf{I}) = \lambda^2 - 3\lambda - 4$ A1

[2]

(e) $\lambda_1 = -1,\ \lambda_2 = 4$ A2

[2]

(f) $\mathbf{v}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix},\ \mathbf{v}_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ A2

[2]

(g)	(i)	$\mathbf{X} = Ae^{-t}\begin{pmatrix}-1\\1\end{pmatrix} + Be^{4t}\begin{pmatrix}4\\1\end{pmatrix}$	(A1) for correct approach
		$\begin{pmatrix}1\\0\end{pmatrix} = Ae^{-0}\begin{pmatrix}-1\\1\end{pmatrix} + Be^{4(0)}\begin{pmatrix}4\\1\end{pmatrix}$	(M1) for substitution
		$\begin{cases}1 = -A + 4B\\ 0 = A + B\end{cases}$	
		By solving this system, $A = -0.2$ and $B = 0.2$.	(A1) for correct values
		$\therefore x = -0.2e^{-t} + 0.2e^{4t}$	A1
	(ii)	The displacement at $t = 0.3$	
		$= -0.2e^{-0.3} + 0.2e^{4(0.3)}$	(M1) for substitution
		$= 0.51585974$	
		$= 0.516$ m	A1
			[6]
(h)	0.113 m		A1
			[1]

4. (a) $\begin{cases} \dfrac{\mathrm{d}v}{\mathrm{d}t} = 9x \\ \dfrac{\mathrm{d}x}{\mathrm{d}t} = v \end{cases}$ A1

[1]

(b) (i) $\begin{cases} v_{n+1} = v_n + 0.25 \left.\dfrac{\mathrm{d}v}{\mathrm{d}t}\right|_{(t_n,\, v_n,\, x_n)} \\ x_{n+1} = x_n + 0.25 \left.\dfrac{\mathrm{d}x}{\mathrm{d}t}\right|_{(t_n,\, v_n,\, x_n)} \\ t_{n+1} = t_n + 0.25 \end{cases}$ (M1) for valid approach

$t_0 = 0,\ v_0 = 0,\ x_0 = 1$ (A1) for correct values

$t_1 = 0 + 0.25 = 0.25$

$v_1 = 0 + 0.25(9) = 2.25$ A1

(ii) $x_1 = 1 + 0.25(0) = 1$ A1

[4]

(c) (i) 1.56 m A1

(ii) 2.69 m A1

(iii) 4.69 m A1

[3]

(d) $\det(\mathbf{M} - \lambda\mathbf{I}) = \begin{vmatrix} 0-\lambda & 9 \\ 1 & 0-\lambda \end{vmatrix}$ (M1) for valid approach

$\det(\mathbf{M} - \lambda\mathbf{I}) = (-\lambda)(-\lambda) - (9)(1)$

$\det(\mathbf{M} - \lambda\mathbf{I}) = \lambda^2 - 9$ A1

[2]

(e) $\lambda_1 = -3,\ \lambda_2 = 3$ A2

[2]

(f) $\mathbf{v}_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix},\ \mathbf{v}_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ A2

[2]

(g) (i) $\mathbf{X} = Ae^{-3t}\begin{pmatrix}-3\\1\end{pmatrix} + Be^{3t}\begin{pmatrix}3\\1\end{pmatrix}$ (A1) for correct approach

$\begin{pmatrix}0\\1\end{pmatrix} = Ae^{-3(0)}\begin{pmatrix}-3\\1\end{pmatrix} + Be^{3(0)}\begin{pmatrix}3\\1\end{pmatrix}$ (M1) for substitution

$\begin{cases} 0 = -3A + 3B \\ 1 = A + B \end{cases}$

By solving this system, $A = 0.5$ and $B = 0.5$. (A1) for correct values

$\therefore x = 0.5e^{-3t} + 0.5e^{3t}$ A1

(ii) The displacement at $t = 1$

$= 0.5e^{-3(1)} + 0.5e^{3(1)}$ (M1) for substitution

$= 10.067662$

$= 10.1\text{ m}$ A1

[6]

(h) The percentage error

$= \left|\dfrac{4.69140625 - 10.067662}{10.067662}\right| \times 100\%$ (A1) for correct substitution

$= 53.40123407\%$

$= 53.4\%$ A1

[2]

Chapter 13 Solution

Exercise 60

1. (a) The characteristic polynomial of $\mathbf{T}$

$= \det(\mathbf{T} - \lambda\mathbf{I})$

$= \begin{vmatrix} 0.05-\lambda & 0.3 \\ 0.95 & 0.7-\lambda \end{vmatrix}$ (M1) for valid approach

$= (0.05-\lambda)(0.7-\lambda) - (0.3)(0.95)$

$= 0.035 - 0.05\lambda - 0.7\lambda + \lambda^2 - 0.285$

$= \lambda^2 - 0.75\lambda - 0.25$ A1

[2]

(b) $\lambda_1 = -\frac{1}{4}$, $\lambda_2 = 1$ A2

[2]

(c) $\mathbf{v}$ is the eigenvector of $\mathbf{T}$ corresponding to $\lambda_2 = 1$. (R1) for correct reasoning

$\therefore \mathbf{v} = \begin{pmatrix} \frac{6}{25} \\ \frac{19}{25} \end{pmatrix}$ A1

[2]

2. (a) The characteristic polynomial of $\mathbf{T}$

$= \det(\mathbf{T}-\lambda\mathbf{I})$

$= \begin{vmatrix} 0.5-\lambda & 0.15 \\ 0.5 & 0.85-\lambda \end{vmatrix}$ (M1) for valid approach

$= (0.5-\lambda)(0.85-\lambda)-(0.15)(0.5)$

$= 0.425-0.5\lambda-0.85\lambda+\lambda^2-0.075$

$= \lambda^2-1.35\lambda+0.35$ A1

[2]

(b) $\lambda_1 = \frac{7}{20}$, $\lambda_2 = 1$ A2

[2]

(c) $\mathbf{v}$ is the eigenvector of $\mathbf{T}$ corresponding to $\lambda_2 = 1$. (R1) for correct reasoning

$\therefore \mathbf{v} = \begin{pmatrix} \frac{3}{13} \\ \frac{10}{13} \end{pmatrix}$ A1

[2]

3. (a) By considering the graph of $y = \det(\mathbf{T}-\lambda\mathbf{I})$,

$\lambda = -\frac{73}{100}$ or $\lambda = 1$. (M1) for valid approach

$\therefore \lambda_1 = -\frac{73}{100}$, $\lambda_2 = 1$ A2

[3]

(b) $\mathbf{v}_8 = \begin{pmatrix} 0.17 & 0.9 \\ 0.83 & 0.1 \end{pmatrix}^8 \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$ (M1) for valid approach

$\mathbf{v}_8 = \begin{pmatrix} 0.5185996472 \\ 0.4814003528 \end{pmatrix}$

$\mathbf{v}_8 = \begin{pmatrix} 0.519 \\ 0.481 \end{pmatrix}$ A1

[2]

(c) $\mathbf{v}$ is the eigenvector of $\mathbf{T}$ corresponding to $\lambda_2 = 1$. (R1) for correct reasoning

$\therefore \mathbf{v} = \begin{pmatrix} \frac{90}{173} \\ \frac{83}{173} \end{pmatrix}$ A1

[2]

4. (a) By considering the graph of $y = \det(\mathbf{T} - \lambda\mathbf{I})$,

$\lambda = -\frac{11}{100}$ or $\lambda = 1$. (M1) for valid approach

$\therefore \lambda_1 = -\frac{11}{100}$, $\lambda_2 = 1$ A2

[3]

(b) $\mathbf{v}_{13} = \begin{pmatrix} 0.52 & 0.63 \\ 0.48 & 0.37 \end{pmatrix}^{13} \begin{pmatrix} 0.09 \\ 0.91 \end{pmatrix}$ (M1) for valid approach

$\mathbf{v}_{13} = \begin{pmatrix} 0.5675675676 \\ 0.4324324324 \end{pmatrix}$

$\mathbf{v}_{13} = \begin{pmatrix} 0.568 \\ 0.432 \end{pmatrix}$ A1

[2]

(c) $\mathbf{v}$ is the eigenvector of $\mathbf{T}$ corresponding to $\lambda_2 = 1$. (R1) for correct reasoning

$\therefore \mathbf{v} = \begin{pmatrix} \frac{21}{37} \\ \frac{16}{37} \end{pmatrix}$ A1

[2]

Exercise 61

1. (a) $\mathbf{T}=\begin{pmatrix}0.7 & 0.25\\ 0.3 & 0.75\end{pmatrix}$ A2

[2]

(b) The characteristic polynomial of $\mathbf{T}$

$=\det(\mathbf{T}-\lambda\mathbf{I})$

$=\begin{vmatrix}0.7-\lambda & 0.25\\ 0.3 & 0.75-\lambda\end{vmatrix}$ (M1) for valid approach

$=(0.7-\lambda)(0.75-\lambda)-(0.25)(0.3)$

$=0.525-0.7\lambda-0.75\lambda+\lambda^2-0.075$

$=\lambda^2-1.45\lambda+0.45$ A1

[2]

(c) $\lambda_1=\frac{9}{20}$, $\lambda_2=1$ A2

[2]

(d) $\mathbf{v}_1=\begin{pmatrix}1\\ -1\end{pmatrix}$, $\mathbf{v}_2=\begin{pmatrix}1\\ \frac{6}{5}\end{pmatrix}$ A2

[2]

(e) (i) $\begin{pmatrix}1 & 1\\ -1 & \frac{6}{5}\end{pmatrix}$ A1

(ii) $\begin{pmatrix}\left(\frac{9}{20}\right)^n & 0\\ 0 & 1\end{pmatrix}$ A2

[3]

(f) $\mathbf{T}^n = \begin{pmatrix} 1 & 1 \\ -1 & \frac{6}{5} \end{pmatrix} \begin{pmatrix} \left(\frac{9}{20}\right)^n & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & \frac{6}{5} \end{pmatrix}^{-1}$

$\mathbf{T}^n = \begin{pmatrix} \left(\frac{9}{20}\right)^n & 1 \\ -\left(\frac{9}{20}\right)^n & \frac{6}{5} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & \frac{6}{5} \end{pmatrix}^{-1}$ A1

$\mathbf{T}^n = \begin{pmatrix} \left(\frac{9}{20}\right)^n & 1 \\ -\left(\frac{9}{20}\right)^n & \frac{6}{5} \end{pmatrix} \begin{pmatrix} \frac{6}{11} & -\frac{5}{11} \\ \frac{5}{11} & \frac{5}{11} \end{pmatrix}$ (A1) for correct approach

$\mathbf{T}^n = \begin{pmatrix} \frac{6}{11}\left(\frac{9}{20}\right)^n + \frac{5}{11} & -\frac{5}{11}\left(\frac{9}{20}\right)^n + \frac{5}{11} \\ -\frac{6}{11}\left(\frac{9}{20}\right)^n + \frac{6}{11} & \frac{5}{11}\left(\frac{9}{20}\right)^n + \frac{6}{11} \end{pmatrix}$ A1

[3]

(g) $\mathbf{T}^5 \begin{pmatrix} 3500 \\ 3500 \end{pmatrix} = \begin{pmatrix} \frac{6\left(\frac{9}{20}\right)^5 + 5}{11} & \frac{-5\left(\frac{9}{20}\right)^5 + 5}{11} \\ \frac{-6\left(\frac{9}{20}\right)^5 + 6}{11} & \frac{5\left(\frac{9}{20}\right)^5 + 6}{11} \end{pmatrix} \begin{pmatrix} 3500 \\ 3500 \end{pmatrix}$ M1A1

$\mathbf{T}^5 \begin{pmatrix} 3500 \\ 3500 \end{pmatrix} = \begin{pmatrix} 3187.689531 \\ 3812.310469 \end{pmatrix}$ (A1) for correct values

Thus, the number of citizens purchasing computers
from American manufacturers after 5 years is 3810. A1

[4]

2. (a) $\mathbf{T}=\begin{pmatrix}0.6 & 0.36\\ 0.4 & 0.64\end{pmatrix}$ A2

[2]

(b) The characteristic polynomial of $\mathbf{T}$

$=\det(\mathbf{T}-\lambda\mathbf{I})$

$=\begin{vmatrix}0.6-\lambda & 0.36\\ 0.4 & 0.64-\lambda\end{vmatrix}$ (M1) for valid approach

$=(0.6-\lambda)(0.64-\lambda)-(0.36)(0.4)$

$=0.384-0.6\lambda-0.64\lambda+\lambda^2-0.144$

$=\lambda^2-1.24\lambda+0.24$ A1

[2]

(c) $\lambda_1=\dfrac{6}{25},\ \lambda_2=1$ A2

[2]

(d) $\mathbf{v}_1=\begin{pmatrix}1\\ -1\end{pmatrix},\ \mathbf{v}_2=\begin{pmatrix}1\\ \dfrac{10}{9}\end{pmatrix}$ A2

[2]

(e) (i) $\begin{pmatrix}1 & 1\\ -1 & \dfrac{10}{9}\end{pmatrix}$ A1

(ii) $\begin{pmatrix}\left(\dfrac{6}{25}\right)^n & 0\\ 0 & 1\end{pmatrix}$ A2

[3]

(f) $$\mathbf{T}^n = \begin{pmatrix} 1 & 1 \\ -1 & \frac{10}{9} \end{pmatrix} \begin{pmatrix} \left(\frac{6}{25}\right)^n & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & \frac{10}{9} \end{pmatrix}^{-1}$$

$$\mathbf{T}^n = \begin{pmatrix} \left(\frac{6}{25}\right)^n & 1 \\ -\left(\frac{6}{25}\right)^n & \frac{10}{9} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & \frac{10}{9} \end{pmatrix}^{-1}$$ A1

$$\mathbf{T}^n = \begin{pmatrix} \left(\frac{6}{25}\right)^n & 1 \\ -\left(\frac{6}{25}\right)^n & \frac{10}{9} \end{pmatrix} \begin{pmatrix} \frac{10}{19} & -\frac{9}{19} \\ \frac{9}{19} & \frac{9}{19} \end{pmatrix}$$ (A1) for correct approach

$$\mathbf{T}^n = \begin{pmatrix} \frac{10}{19}\left(\frac{6}{25}\right)^n + \frac{9}{19} & -\frac{9}{19}\left(\frac{6}{25}\right)^n + \frac{9}{19} \\ -\frac{10}{19}\left(\frac{6}{25}\right)^n + \frac{10}{19} & \frac{9}{19}\left(\frac{6}{25}\right)^n + \frac{10}{19} \end{pmatrix}$$ A1

[3]

(g) $$\mathbf{T}^7 \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{10}{19}\left(\frac{6}{25}\right)^7 + \frac{9}{19} & -\frac{9}{19}\left(\frac{6}{25}\right)^7 + \frac{9}{19} \\ -\frac{10}{19}\left(\frac{6}{25}\right)^7 + \frac{10}{19} & \frac{9}{19}\left(\frac{6}{25}\right)^7 + \frac{10}{19} \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$$ M1A1

$$\mathbf{T}^7 \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 0.4736930616 \\ 0.5263069384 \end{pmatrix}$$ (A1) for correct values

Thus, the probability that it is cloudy after a week is 0.526. A1

[4]

3. (a) $\mathbf{T}=\begin{pmatrix}0.82 & 0.13\\ 0.18 & 0.87\end{pmatrix}$ A2

[2]

(b) By considering the graph of $y=\det(\mathbf{T}-\lambda\mathbf{I})$,

$\lambda=\frac{69}{100}$ or $\lambda=1$. (M1) for valid approach

$\therefore \lambda_1=\frac{69}{100}$, $\lambda_2=1$ A2

[3]

(c) $\mathbf{v}_1=\begin{pmatrix}1\\ -1\end{pmatrix}$, $\mathbf{v}_2=\begin{pmatrix}1\\ \frac{18}{13}\end{pmatrix}$ A2

[2]

(d) (i) $\begin{pmatrix}1 & 1\\ -1 & \frac{18}{13}\end{pmatrix}$ A1

(ii) $\begin{pmatrix}\left(\frac{69}{100}\right)^n & 0\\ 0 & 1\end{pmatrix}$ A2

[3]

(e) $\mathbf{T}^n=\begin{pmatrix}1 & 1\\ -1 & \frac{18}{13}\end{pmatrix}\begin{pmatrix}\left(\frac{69}{100}\right)^n & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ -1 & \frac{18}{13}\end{pmatrix}^{-1}$

$\mathbf{T}^n=\begin{pmatrix}\left(\frac{69}{100}\right)^n & 1\\ -\left(\frac{69}{100}\right)^n & \frac{18}{13}\end{pmatrix}\begin{pmatrix}1 & 1\\ -1 & \frac{18}{13}\end{pmatrix}^{-1}$ A1

$\mathbf{T}^n=\begin{pmatrix}\left(\frac{69}{100}\right)^n & 1\\ -\left(\frac{69}{100}\right)^n & \frac{18}{13}\end{pmatrix}\begin{pmatrix}\frac{18}{31} & -\frac{13}{31}\\ \frac{13}{31} & \frac{13}{31}\end{pmatrix}$ (A1) for correct approach

$\mathbf{T}^n=\begin{pmatrix}\frac{18}{31}\left(\frac{69}{100}\right)^n+\frac{13}{31} & -\frac{13}{31}\left(\frac{69}{100}\right)^n+\frac{13}{31}\\ -\frac{18}{31}\left(\frac{69}{100}\right)^n+\frac{18}{31} & \frac{13}{31}\left(\frac{69}{100}\right)^n+\frac{18}{31}\end{pmatrix}$ A1

[3]

(f) $\mathbf{T}^n\begin{pmatrix}310\\310\end{pmatrix}=\begin{pmatrix}50\left(\frac{69}{100}\right)^n+260\\-50\left(\frac{69}{100}\right)^n+360\end{pmatrix}$ A2

[2]

(g) (i) 260 A1

(ii) 360 A1

[2]

4. (a) $\mathbf{T}=\begin{pmatrix}0.9 & 0.07\\ 0.1 & 0.93\end{pmatrix}$ A2

[2]

(b) By considering the graph of $y=\det(\mathbf{T}-\lambda\mathbf{I})$,

$\lambda=\frac{83}{100}$ or $\lambda=1$. (M1) for valid approach

$\therefore \lambda_1=\frac{83}{100}$, $\lambda_2=1$ A2

[3]

(c) $\mathbf{v}_1=\begin{pmatrix}1\\ -1\end{pmatrix}$, $\mathbf{v}_2=\begin{pmatrix}1\\ \frac{10}{7}\end{pmatrix}$ A2

[2]

(d) (i) $\begin{pmatrix}1 & 1\\ -1 & \frac{10}{7}\end{pmatrix}$ A1

(ii) $\begin{pmatrix}\left(\frac{83}{100}\right)^n & 0\\ 0 & 1\end{pmatrix}$ A2

[3]

(e) $$\mathbf{T}^n=\begin{pmatrix}1 & 1\\ -1 & \frac{10}{7}\end{pmatrix}\begin{pmatrix}\left(\frac{83}{100}\right)^n & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ -1 & \frac{10}{7}\end{pmatrix}^{-1}$$

$$\mathbf{T}^n=\begin{pmatrix}\left(\frac{83}{100}\right)^n & 1\\ -\left(\frac{83}{100}\right)^n & \frac{10}{7}\end{pmatrix}\begin{pmatrix}1 & 1\\ -1 & \frac{10}{7}\end{pmatrix}^{-1}$$ A1

$$\mathbf{T}^n=\begin{pmatrix}\left(\frac{83}{100}\right)^n & 1\\ -\left(\frac{83}{100}\right)^n & \frac{10}{7}\end{pmatrix}\begin{pmatrix}\frac{10}{17} & -\frac{7}{17}\\ \frac{7}{17} & \frac{7}{17}\end{pmatrix}$$ (A1) for correct approach

$$\mathbf{T}^n=\begin{pmatrix}\frac{10}{17}\left(\frac{83}{100}\right)^n+\frac{7}{17} & -\frac{7}{17}\left(\frac{83}{100}\right)^n+\frac{7}{17}\\ -\frac{10}{17}\left(\frac{83}{100}\right)^n+\frac{10}{17} & \frac{7}{17}\left(\frac{83}{100}\right)^n+\frac{10}{17}\end{pmatrix}$$ A1

[3]

(f) $\mathbf{T}^n \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix} = \begin{pmatrix} \frac{16}{85}\left(\frac{83}{100}\right)^n + \frac{7}{17} \\ -\frac{16}{85}\left(\frac{83}{100}\right)^n + \frac{10}{17} \end{pmatrix}$ A2

[2]

(g) The proportion of the supporters of A in 2013

$= \frac{16}{85}\left(\frac{83}{100}\right)^{-6} + \frac{7}{17}$ (A1) for substitution

$= 0.9875127916$

$= 0.988$ A1

[2]

(h) (i) $\frac{7}{17}$ A1

(ii) $\frac{10}{17}$ A1

[2]

Chapter 14 Solution

Exercise 62

1. (a) $X \sim \text{Po}(1.7)$

$\text{P}(X < 2) = \text{P}(X \le 1)$ (M1) for valid approach

$\text{P}(X < 2) = 0.4932455154$

$\text{P}(X < 2) = 0.493$ A1

[2]

(b) $Y \sim \text{Po}(10.2)$ (M1) for valid approach

$\text{P}(Y = 10) = 0.1248633462$

$\text{P}(Y = 10) = 0.125$ A1

[2]

(c) $\text{P}(Y < 6 \mid Y < 9) = \dfrac{\text{P}(Y < 6 \cap Y < 9)}{\text{P}(Y < 9)}$ (A1) for substitution

$\text{P}(Y < 6 \mid Y < 9) = \dfrac{\text{P}(Y \le 5)}{\text{P}(Y \le 8)}$

$\text{P}(Y < 6 \mid Y < 9) = \dfrac{0.0598876554}{0.3107558583}$ (A1) for correct approach

$\text{P}(Y < 6 \mid Y < 9) = 0.1927160946$

$\text{P}(Y < 6 \mid Y < 9) = 0.193$ A1

[3]

2. (a) $X \sim \text{Po}(12.8)$

$\text{P}(X > 10) = 1 - \text{P}(X \leq 10)$ (M1) for valid approach

$\text{P}(X > 10) = 1 - 0.2692514839$

$\text{P}(X > 10) = 0.7307485161$

$\text{P}(X > 10) = 0.731$ A1

[2]

(b) $Y \sim \text{Po}(38.4)$ (M1) for valid approach

$\text{P}(Y \leq 33) = 0.2174266935$

$\text{P}(Y \leq 33) = 0.217$ A1

[2]

(c) $\text{P}(Y = 43 \mid Y \geq 40) = \dfrac{\text{P}(Y = 43 \cap Y \geq 40)}{\text{P}(Y \geq 40)}$ (A1) for substitution

$\text{P}(Y = 43 \mid Y \geq 40) = \dfrac{\text{P}(Y = 43)}{1 - \text{P}(Y \leq 39)}$

$\text{P}(Y = 43 \mid Y \geq 40) = \dfrac{0.0465787545}{1 - 0.580636828}$ (A1) for correct approach

$\text{P}(Y = 43 \mid Y \geq 40) = 0.1110702074$

$\text{P}(Y = 43 \mid Y \geq 40) = 0.111$ A1

[3]

3. (a) $X \sim \text{Po}(3.1)$

$\text{P}(X \geq 2) = 1 - \text{P}(X \leq 1)$ (M1) for valid approach

$\text{P}(X \geq 2) = 1 - 0.1847017298$

$\text{P}(X \geq 2) = 0.8152982702$

$\text{P}(X \geq 2) = 0.815$ A1

[2]

(b) $Y \sim \text{Po}(21.7)$ (M1) for valid approach

$\text{P}(Y \leq 28) = 0.9231077265$

$\text{P}(Y \leq 28) = 0.923$ A1

[2]

(c) $\text{P}(X \leq 4)^7 = 0.7981894641^7$ (M1) for valid approach

$\text{P}(X \leq 4)^7 = 0.2064153244$

$\text{P}(X \leq 4)^7 = 0.206$ A1

[2]

4. (a) $X \sim \text{Po}(7)$

$\text{P}(X > 7) = 1 - \text{P}(X \leq 7)$ (M1) for valid approach

$\text{P}(X > 7) = 1 - 0.598713836$

$\text{P}(X > 7) = 0.401286164$

$\text{P}(X > 7) = 0.401$ A1

[2]

(b) $X_0 \sim \text{Po}(1.75)$ (M1) for valid approach

$\text{P}(X_0 \leq 2) = 0.7439696955$

$\text{P}(X_0 \leq 2) = 0.744$ A1

[2]

(c) $Y \sim \text{Po}(6)$

$\text{P}(X = 5)\text{P}(Y = 5) = (0.1277166683)(0.160623141)$ (M1) for valid approach

$\text{P}(X = 5)\text{P}(Y = 5) = 0.0205142524$

$\text{P}(X = 5)\text{P}(Y = 5) = 0.0205$ A1

[2]

Exercise 63

1. (a) $P(X=0)=\frac{3}{32}P(X=3)$

$\therefore \frac{e^{-b}\cdot b^0}{0!}=\frac{3}{32}\left(\frac{e^{-b}\cdot b^3}{3!}\right)$ (M1) for setting equation

$e^{-b}=\frac{b^3 e^{-b}}{64}$ (A1) for correct approach

$b^3=64$

$b=4$ A1

[3]

(b) (i) $P(X=2)=0.1465251111$

$P(X=2)=0.147$ A1

(ii) 4 A1

[2]

2. (a) $9P(X=1)=2P(X=2)$

$\therefore 9\left(\frac{e^{-c}\cdot c^1}{1!}\right)=2\left(\frac{e^{-c}\cdot c^2}{2!}\right)$ (M1) for setting equation

$9ce^{-c}=c^2e^{-c}$ (A1) for correct approach

$c=9$ A1

[3]

(b) (i) $P(X\le 4)=0.0549636415$

$P(X\le 4)=0.0550$ A1

(ii) 3 A1

[2]

3. (a) $X \sim \text{Po}(\lambda)$

$\text{P}(X = 13) = 0.0956$

$\therefore \dfrac{e^{-\lambda} \cdot \lambda^{13}}{13!} = 0.0956$ (M1) for setting equation

$\dfrac{e^{-\lambda} \cdot \lambda^{13}}{6227020800} - 0.0956 = 0$ (A1) for correct approach

By considering the graph of

$y = \dfrac{e^{-\lambda} \cdot \lambda^{13}}{6227020800} - 0.0956$, $\lambda = 15.000534$.

$\therefore \lambda = 15$ A1

[3]

(b) $Y \sim \text{Po}\left(\dfrac{15}{7}\right)$

$\text{P}(Y > 3) = 1 - \text{P}(Y \le 3)$ (M1) for valid approach

$\text{P}(Y > 3) = 1 - 0.8304691992$

$\text{P}(Y > 3) = 0.1695308008$

$\text{P}(Y > 3) = 0.170$ A1

[2]

4. (a) $X \sim \text{Po}(\lambda)$

$\text{P}(X = 1) = 0.00990$

$\therefore \dfrac{e^{-\lambda} \cdot \lambda^{1}}{1!} = 0.00990$ (M1) for setting equation

$\lambda e^{-\lambda} - 0.00990 = 0$ (A1) for correct approach

By considering the graph of $y = \lambda e^{-\lambda} - 0.00990$,

$\lambda = 0.0099995$.

$\therefore \lambda = 0.0100$ A1

[3]

(b) $Y \sim \text{Po}(1.19994)$

$\text{P}(Y > 2) = 1 - \text{P}(Y \le 2)$ (M1) for valid approach

$\text{P}(Y > 2) = 1 - 0.8795001102$

$\text{P}(Y > 2) = 0.1204998898$

$\text{P}(Y > 2) = 0.120$ A1

[2]

Chapter 15 Solution

Exercise 64

1. (a) $E(1-10X)=1-10E(X)$

$E(1-10X)=1-10(21.5)$ (A1) for substitution

$E(1-10X)=-214$ A1

[2]

(b) $Var(5+2X)=2^2Var(X)$

$Var(5+2X)=2^2(3)$ (A1) for substitution

$Var(5+2X)=12$ A1

[2]

(c) $E(5+4X+3Y)=5+4E(X)+3E(Y)$

$E(5+4X+3Y)=5+4(21.5)+3(20)$ (A1) for substitution

$E(5+4X+3Y)=151$ A1

[2]

2. (a) $E(5X)=5E(X)$

$40=5E(X)$ (A1) for correct equation

$E(X)=8$ A1

[2]

(b) $Var(1+2X)=2^2Var(X)$

$4=4Var(X)$ (A1) for correct equation

$Var(X)=1$ A1

[2]

(c) $E(-X-Y)=-E(X)-E(Y)$

$E(-X-Y)=-8-20$ (A1) for substitution

$E(-X-Y)=-28$ A1

[2]

3. (a) $E(100 - X) = 100 - E(X)$

$E(100 - X) = 100 - (-5)$ (A1) for substitution

$E(100 - X) = 105$ A1

[2]

(b) $\text{Var}(99 - 5X) = (-5)^2 \text{Var}(X)$

$\text{Var}(99 - 5X) = 25(16)$ (A1) for substitution

$\text{Var}(99 - 5X) = 400$ A1

[2]

(c) $\text{Var}(6X - 5Y) = 6^2 \text{Var}(X) + (-5)^2 \text{Var}(Y)$

$\text{Var}(6X - 5Y) = 36(16) + 25(8)$ (A1) for substitution

$\text{Var}(6X - 5Y) = 776$ A1

[2]

4. (a) $E(8 - 7X) = 8 - 7E(X)$

$29 = 8 - 7E(X)$ (A1) for correct equation

$21 = -7E(X)$

$E(X) = -3$ A1

[2]

(b) $\text{Var}(-7X) = (-7)^2 \text{Var}(X)$

$147 = 49\text{Var}(X)$ (A1) for correct equation

$\text{Var}(X) = 3$ A1

[2]

(c) $\text{Var}(10Y - 3X) = 10^2 \text{Var}(Y) + (-3)^2 \text{Var}(X)$

$\text{Var}(10Y - 3X) = 100(4.5) + 9(3)$ (A1) for substitution

$\text{Var}(10Y - 3X) = 477$ A1

[2]

Exercise 65

1.	(a)	(i)	640 g	A1	
		(ii)	256 g	A1	
					[2]
	(b)	(i)	760 g	A1	
		(ii)	20.4 g	A1	
					[2]
	(c)		$Y \sim \text{N}(760, 416)$		
			$\text{P}(Y < 795) = 0.9569204948$	(A1) for correct value	
			$\text{P}(Y < 795) = 0.957$	A1	
					[2]
2.	(a)	(i)	1068 cm	A1	
		(ii)	54 cm	A1	
					[2]
	(b)		$\text{P}(1070 < X < 1090) = 0.391369825$	(A1) for correct value	
			$\text{P}(1070 < X < 1090) = 0.391$	A1	
					[2]
	(c)		$Y \sim \text{N}(343, 25)$	(A1) for correct approach	
			$\text{P}(Y > 330) = 0.9953387782$	(A1) for correct value	
			$\text{P}(Y > 330) = 0.995$	A1	
					[3]
3.	(a)		$X + Y \sim \text{N}(970, 458)$	(A1) for correct approach	
			$\text{P}(X + Y > 1000) = 0.0804863549$	(A1) for correct value	
			$\text{P}(X + Y > 1000) = 0.0805$	A1	
					[3]
	(b)		$X - Y \sim \text{N}(30, 458)$	(A1) for correct approach	
			$\text{P}(X - Y > 0) = 0.9195136451$	(A1) for correct value	
			$\text{P}(X - Y > 0) = 0.920$	A1	
					[3]

4. (a) $X+Y \sim \mathrm{N}(17, 2.8125)$ (A1) for correct approach

$\mathrm{P}(16.2 < X+Y < 17.8) = 0.3666576874$ (A1) for correct value

$\mathrm{P}(16.2 < X+Y < 17.8) = 0.367$ A1

[3]

(b) $X-Y \sim \mathrm{N}(3, 2.8125)$ (A1) for correct approach

$\mathrm{P}(X-Y < 0) = 0.0368190835$ (A1) for correct value

$\mathrm{P}(X-Y < 0) = 0.0368$ A1

[3]

Exercise 66

1. (a) $\mathrm{E}(X+Y)=\mathrm{E}(X)+\mathrm{E}(Y)$

$\mathrm{E}(X+Y)=5.5+3.5$ (A1) for substitution

$\mathrm{E}(X+Y)=9$ A1

[2]

(b) $\mathrm{P}(5<X+Y<8)=\mathrm{P}(X+Y=6)+\mathrm{P}(X+Y=7)$ (M1) for valid approach

$\mathrm{P}(5<X+Y<8)=0.091090319+0.1171161245$

$\mathrm{P}(5<X+Y<8)=0.2082064435$

$\mathrm{P}(5<X+Y<8)=0.208$ A1

[2]

(c) $\mathrm{Var}(X-4Y)=\mathrm{Var}(X)+(-4)^2\,\mathrm{Var}(Y)$

$\mathrm{Var}(X-4Y)=5.5+(16)(3.5)$ (A1) for substitution

$\mathrm{Var}(X-4Y)=61.5$ A1

[2]

2. (a) $\mathrm{E}(W+X+Y)=\mathrm{E}(W)+\mathrm{E}(X)+\mathrm{E}(Y)$

$\mathrm{E}(W+X+Y)=12+18+4$ (A1) for substitution

$\mathrm{E}(W+X+Y)=34$ A1

[2]

(b) $\mathrm{P}(W+X+Y\geq 30)=1-\mathrm{P}(W+X+Y\leq 29)$ (M1) for valid approach

$\mathrm{P}(W+X+Y\geq 30)=1-0.2235048732$

$\mathrm{P}(W+X+Y\geq 30)=0.7764951268$

$\mathrm{P}(W+X+Y\geq 30)=0.776$ A1

[2]

(c) $\mathrm{Var}(3W-2X-Y)$

$=3^2\,\mathrm{Var}(W)+(-2)^2\,\mathrm{Var}(X)+(-1)^2\,\mathrm{Var}(Y)$

$\mathrm{Var}(3W-2X-Y)=9(12)+4(18)+4$ (A1) for substitution

$\mathrm{Var}(3W-2X-Y)=184$ A1

[2]

3. (a) Let X and Y be the number of hamburgers and onion ring boxes sold in a particular hour respectively.

The expected total number

$= \mathrm{E}(X+Y)$

$= \mathrm{E}(X)+\mathrm{E}(Y)$

$= 24+15.5$ (A1) for substitution

$= 39.5$ A1

[2]

(b) The required probability

$= \mathrm{P}(X+Y=38)$ (M1) for valid approach

$= 0.0627389331$

$= 0.0627$ A1

[2]

(c) The required probability

$= \mathrm{P}(X+Y=38)^6$ (M1) for valid approach

$= 0.0627389331^6$

$= 6.098496438\times10^{-8}$

$= 6.10\times10^{-8}$ A1

[2]

4. (a) Let X and Y be the number of views of the football video and the basketball video on a particular day respectively.

The expected total number

$= \mathrm{E}(X+Y)$

$= \mathrm{E}(X)+\mathrm{E}(Y)$

$= 195+180$ (A1) for substitution

$= 375$ A1

[2]

(b) The required probability

$= \mathrm{P}(X+Y=376)$ (M1) for valid approach

$= 0.0205419347$

$= 0.0205$ A1

[2]

(c) The required probability

$= \mathrm{P}(X+Y=376)^3$ (M1) for valid approach

$= 0.0205419347^3$

$= 8.668102396\times10^{-6}$

$= 8.67\times10^{-6}$ A1

[2]

Exercise 67

1. (a) $E(X) = (2)(0.6) + (4)(0.1) + (6)(0.15) + (8)(0.15)$ (A1) for substitution

$E(X) = 3.7$ A1

[2]

(b) $E(5X + 2Y) = 5E(X) + 2E(Y)$

$E(5X + 2Y) = 5(3.7) + 2(10)$ (A1) for substitution

$E(5X + 2Y) = 38.5$ A1

[2]

(c) $\text{Var}(5X + 2Y) = 5^2 \text{Var}(X) + 2^2 \text{Var}(Y)$

$\text{Var}(5X + 2Y) = 25(5.31) + 4(2)$ (A1) for substitution

$\text{Var}(5X + 2Y) = 140.75$ A1

[2]

2. (a) $E(X) = (100)(0.65)$ (A1) for substitution

$E(X) = 65$ A1

[2]

(b) $\text{Var}(X) = (100)(0.65)(1 - 0.65)$ (A1) for substitution

$\text{Var}(X) = 22.75$ A1

[2]

(c) $E(2X - 7Y) = 2E(X) - 7E(Y)$

$E(2X - 7Y) = 2(65) - 7(30)$ (A1) for substitution

$E(2X - 7Y) = -80$ A1

[2]

(d) $\text{Var}(2X - 7Y) = 2^2 \text{Var}(X) + (-7)^2 \text{Var}(Y)$

$\text{Var}(2X - 7Y) = 4(22.75) + 49(7)$ (A1) for substitution

$\text{Var}(2X - 7Y) = 434$ A1

[2]

3. (a) $E(X) = (15)(0.03)$ (A1) for substitution

$E(X) = 0.45$ A1

[2]

(b) $Var(X) = (15)(0.03)(1-0.03)$ (A1) for substitution

$Var(X) = 0.4365$ A1

[2]

(c) $E(-X-Y) = -E(X) - E(Y)$

$E(-X-Y) = -0.45 - (-1.2)$ (A1) for substitution

$E(-X-Y) = 0.75$ A1

[2]

(d) $Var(-X-Y) = (-1)^2 Var(X) + (-1)^2 Var(Y)$

$Var(-X-Y) = 0.4365 + 0.8$ (A1) for substitution

$Var(-X-Y) = 1.2365$ A1

[2]

4. (a) (i) $\frac{1}{5}$ A1

(ii) $P(X=0) = 1 - \frac{1}{5} - \left(\frac{4}{5}\right)\left(\frac{1}{4}\right)$ (A1) for substitution

$P(X=0) = \frac{3}{5}$ A1

(iii) $E(X) = (20)\left(\frac{1}{5}\right) + (10)\left(\frac{4}{5}\right)\left(\frac{1}{4}\right) + (0)\left(\frac{16}{25}\right)$ (A1) for substitution

$E(X) = 6$ A1

[5]

(b) $E(X+6Y) = E(X) + 6E(Y)$

$8 = 6 + 6E(Y)$ (A1) for substitution

$2 = 6E(Y)$

$E(Y) = \frac{1}{3}$ A1

[2]

Chapter 16 Solution

Exercise 68

1. (a) 300 — A1

[1]

(b) $\text{Var}(\bar{X}) = \frac{\text{Var}(X)}{n}$

$\text{Var}(\bar{X}) = \frac{4.5}{180}$ — (A1) for substitution

$\text{Var}(\bar{X}) = \frac{1}{40}$ — A1

[2]

(c) $\bar{X} \sim \text{N}\left(300, \frac{1}{40}\right)$ — (M1) for valid approach

$\text{P}(\bar{X} < 299.85) = 0.1713908408$

$\text{P}(\bar{X} < 299.85) = 0.171$ — A1

[2]

2. (a) -2 — A1

[1]

(b) The standard deviation of $\bar{X}$

$= \sqrt{\frac{\text{Var}(X)}{n}}$

$= \sqrt{\frac{8}{32}}$ — (A1) for substitution

$= \frac{1}{2}$ — A1

[2]

(c) $\bar{X} \sim \text{N}\left(-2, \left(\frac{1}{2}\right)^2\right)$ — (M1) for valid approach

$\text{P}(|\bar{X}| < 1.5) = \text{P}(-1.5 < \bar{X} < 1.5)$ — (A1) for correct approach

$\text{P}(|\bar{X}| < 1.5) = 0.1586552596$

$\text{P}(|\bar{X}| < 1.5) = 0.159$ — A1

[3]

3. (a) 5 A1

[1]

(b) $\text{Var}(\bar{X}) = \dfrac{\text{Var}(X)}{n}$

$\text{Var}(\bar{X}) = \dfrac{1.6}{50}$ (A1) for substitution

$\text{Var}(\bar{X}) = \dfrac{4}{125}$ A1

[2]

(c) $\bar{X} + \bar{Y} \sim \text{N}\left(5 + (-5), \dfrac{4}{125} + \dfrac{0.8}{50}\right)$ (M2) for valid approach

$\text{P}(\bar{X} + \bar{Y} > 0.1) = 0.3240384511$

$\text{P}(\bar{X} + \bar{Y} > 0.1) = 0.324$ A1

[3]

4. (a) $\text{Var}(\bar{X}) = \dfrac{\text{Var}(X)}{n}$

$\left(\dfrac{1}{3}\right)^2 = \dfrac{12}{n}$ (M1) for setting equation

$n = 108$ A1

[2]

(b) $\bar{X} - \bar{Y} \sim \text{N}\left(180 - 160, \left(\dfrac{1}{3}\right)^2 + \dfrac{48}{216}\right)$ (M2) for valid approach

$\text{P}(9 < \bar{X} - \bar{Y} < 21) = 0.9583677808$

$\text{P}(9 < \bar{X} - \bar{Y} < 21) = 0.958$ A1

[3]

Exercise 69

1. (a) $s_{n-1} = \sqrt{\frac{n}{n-1}} s_n$

$s_{n-1} = \sqrt{\frac{30}{30-1}}(5)$ (A1) for substitution

$s_{n-1} = 5.085476277$

$s_{n-1} = 5.09$ A1

[2]

(b) An unbiased estimate

$= \bar{X}$ (A1) for correct approach

$= \frac{1770}{30}$

$= 59$ A1

[2]

2. (a) $s_{n-1}^2 = \frac{n}{n-1} s_n^2$

$1.25 = \frac{25}{25-1} s_n^2$ (M1) for setting equation

$s_n^2 = 1.2$ A1

[2]

(b) The total number of complaint calls

$= (7.8)(25)$ (A1) for correct approach

$= 195$ A1

[2]

3. (a) $s_{n-1} = \sqrt{\frac{n}{n-1}} s_n$

$2.37 = \sqrt{\frac{60}{60-1}} s_n$ (M1) for setting equation

$s_n = 2.350167015$

$s_n = 2.35$ A1

[2]

(b) The total sum of numbers

$= (-3.8)(60)$ (A1) for correct approach

$= -228$ A1

[2]

4. (a) (i) 16.6 A1

(ii) 3.3 A1

[2]

(b) $s_{n-1}^2 = \frac{n}{n-1} s_n^2$

$3.3 = \frac{n}{n-1}(3.27)$ (M1) for setting equation

$3.3(n-1) = 3.27n$

$3.3n - 3.3 = 3.27n$ (A1) for correct approach

$0.03n = 3.3$

$n = 110$ A1

[3]

Chapter 17 Solution

Exercise 70

1. (a) An unbiased estimate

$$= \frac{300 + 302 + \cdots + 491}{12}$$ (A1) for correct approach

$= 401\text{ g}$ A1

[2]

(b) $$s_{n-1} = \sqrt{\frac{(300-401)^2 + (302-401)^2 + \cdots + (491-401)^2}{12-1}}$$ (A1) for correct approach

$s_{n-1} = 103.3607979\text{ g}$

$s_{n-1} = 103\text{ g}$ A1

[2]

(c) 99% confidence interval:

(308.33, 493.67) A2

[2]

2. (a) An unbiased estimate

$$= \frac{66 + 50 + 34 + 51 + 71 + 52}{6}$$ (A1) for correct approach

$= 54\text{ s}$ A1

[2]

(b) 95% confidence interval:

(43.442, 64.558) A2

[2]

3. (a) $n = \frac{500}{20}$ (M1) for valid approach

$n = 25$ A1

[2]

(b) 90% confidence interval:

(19.013, 20.987) A2

[2]

(c) 1.974 A1

[1]

4. (a) $s_{n-1}^2 = \frac{n}{n-1} s_n^2$

$\frac{16}{15} s_n^2 = \frac{n}{n-1} s_n^2$ (M1) for setting equation

$\frac{16}{15} = \frac{n}{n-1}$

$16(n-1) = 15n$ (A1) for correct approach

$16n - 16 = 15n$

$n = 16$ A1

[3]

(b) 90% confidence interval:
(68.685, 71.315) A2

[2]

(c) 2.63 A1

[1]

Exercise 71

1. (a) An unbiased estimate

$= \bar{X}$ (A1) for correct approach

$= \dfrac{18.25 + 23.05}{2}$

$= 20.65$ A1

[2]

(b) $23.05 - 18.25 = 2(2.575829303)\left(\dfrac{\sigma}{\sqrt{8}}\right)$ M1A1

$\sigma = 2.635355181$

$\sigma = 2.64$ A1

[3]

2. (a) An unbiased estimate

$= \bar{X}$ (A1) for correct approach

$= \dfrac{2.995 + 3.365}{2}$

$= 3.18$ A1

[2]

(b) $3.365 - 2.995 = 2(2.063898542)\left(\dfrac{s_{n-1}}{\sqrt{25}}\right)$ M1A1

$s_{n-1} = 0.4481809455$

$s_{n-1} = 0.448$ A1

[3]

3. (a) A confidence interval with a smaller confidence level has a narrower interval about the mean. R1

[1]

(b) (36.4, 38.6) A1

[1]

(c) $2.2 = 2(2.131449536)\left(\dfrac{s_{n-1}}{\sqrt{16}}\right)$ M1A1

$s_{n-1} = 2.064322859$

$s_{n-1} = 2.06$ A1

[3]

4. (a) A confidence interval with a smaller confidence level has a narrower interval about the mean. R1

[1]

(b) (191.6, 204.4) A1

[1]

(c) $12.8 = 2(2.575829303)\left(\frac{\sigma}{\sqrt{9}}\right)$ M1A1

$\sigma = 7.453910078$

$\sigma = 7.45$ A1

[3]

Chapter 18 Solution

Exercise 72

1. (a) (i) $a = 6225.806452$

$a = 6226$ A1

$b = 2677.419355$

$b = 2677$ A1

(ii) The estimated distance travelled

$= 6225.806452(4.5) + 2677.419355$ (A1) for substitution

$= 30693.54839$ km

$= 30700$ km A1

[4]

(b) (i) $r = 0.8649752713$

$r = 0.865$ A1

(ii) $R^2 = 0.7481822199$

$R^2 = 0.748$ A1

(iii) 74.8% of the variability of the data is explained by the regression model. A1

[3]

2. (a) (i) $a = -0.1319884726$

$a = -0.132$ A1

$b = 8.093371758$

$b = 8.09$ A1

(ii) The estimated pure weight

$= -0.1319884726(25) + 8.093371758$ (A1) for substitution

$= 4.793659943$ g

$= 4.79$ g A1

[4]

(b) (i) $r = -0.7728771798$

$r = -0.773$ A1

(ii) $R^2 = 0.597339135$

$R^2 = 0.597$ A1

(iii) 59.7% of the variability of the data is explained by the regression model. A1

[3]

3. (a) (i) $a = 0.5327664869$

$a = 0.533$ A1

$b = 13.86126089$

$b = 13.9$ A1

(ii) a represents the increase in expenditure when the time spent in a market increases by 1 minute. A1

[3]

(b) (i) $r = 0.4788903591$

$r = 0.479$ A1

(ii) $R^2 = 0.229335976$

$R^2 = 0.229$ A1

(iii) 22.9% of the variability of the data is explained by the regression model. A1

[3]

4.	(a)	(i)	$a = 0.11$	A1
			$b = -0.26$	A1
		(ii)	b represents the annual corn yield rate when the annual rainfall is 0 cm.	A1
				[3]
	(b)	(i)	$r = 0.9574271078$	
			$r = 0.957$	A1
		(ii)	$R^2 = 0.9166666667$	
			$R^2 = 0.917$	A1
		(iii)	91.7% of the variability of the data is explained by the regression model.	A1
				[3]

Exercise 73

1. (a) (i) 1050 USD A1

(ii) 810 USD A1

(iii) 450 USD A1

[3]

(b) $SS_{res} = (1050-1100)^2 + ((-400(0.5)+1130)-900)^2$

$+(810-850)^2 + ((-400(1.2)+1130)-500)^2$ (A1) for correct approach

$+(450-530)^2$

$SS_{res} = 33900$ A1

[2]

(c) 87.2% of the variability of the data is explained by the regression model. A1

[1]

2. (a) (i) 74.6625 marks A1

(ii) 78.9625 marks A1

(iii) 74.0625 marks A1

[3]

(b) $SS_{res} = ((-1.15(1.5)^2 + 12.5(1.5)+45)-60)^2$

$+(74.6625-75)^2 + (-1.15(5)^2 + 12.5(5)+45-85)^2$ (A1) for correct approach

$+(78.9625-75)^2 + (-1.15(6)^2 + 12.5(6)+45-70)^2$

$+(74.0625-75)^2$

$SS_{res} = 131.068125$

$SS_{res} = 131$ A1

[2]

(c) 63.2% of the variability of the data is explained by the regression model. A1

[1]

3. (a) (i) 9231 A1

(ii) 7200 A1

(iii) 11264 A1

[3]

(b) $SS_{res} = (6800 - 6700)^2 + (7200 - 7400)^2$
$+ (7600 - 7500)^2$ (A1) for correct approach

$SS_{res} = 60000$ A1

[2]

(c) Model 1 A1

[1]

4. (a) (i) 212 A1

(ii) 192 A1

(iii) 155 A1

[3]

(b) $SS_{res} = (100 \cdot 1.02^{26} - 140)^2 + (100 \cdot 1.02^{24} - 130)^2$
$+ (100 \cdot 1.02^{22} - 135)^2$ (A1) for correct approach

$SS_{res} = 2082.990335$

$SS_{res} = 2080$ A1

[2]

(c) Model 2 A1

[1]

Exercise 74

1.	(a)	(i)	$y = 0.00867x^3 - 3.92x^2 + 590x - 29500$	A2	
		(ii)	The estimated weight $= 0.0086666667(148)^3 - 3.92(148)^2$ $+590.2833333(148) - 29533.9$ $= 59.88410313$ kg $= 59.9$ kg	(A1) for substitution A1	
					[4]
	(b)	(i)	$R^2 = 0.9669811321$ $R^2 = 0.967$	A1	
		(ii)	96.7% of the variability of the data is explained by the regression model.	A1	
					[2]
2.	(a)	(i)	$y = 23.8x^{0.169}$	A2	
		(ii)	The estimated weight $= 23.82365751(1.25)^{0.1691472196}$ $= 24.74004563$ g $= 24.7$ g	(A1) for substitution A1	
					[4]
	(b)	(i)	$R^2 = 0.3601533129$ $R^2 = 0.360$	A1	
		(ii)	36.0% of the variability of the data is explained by the regression model.	A1	
					[2]

3. (a) (i) $y = 0.25x^2 + 2.25x - 1$ A2

(ii) $R^2 = 0.9666666667$

$R^2 = 0.967$ A1

[3]

(b) $SS_{res} = 1$ A2

[2]

(c) $R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$

$0.9666666667 = 1 - \frac{1}{SS_{tot}}$ (A1) for substitution

$\frac{1}{SS_{tot}} = 0.0333333333$

$SS_{tot} = 30.00000003$

$SS_{tot} = 30.0$ A1

[2]

4. (a) (i) $y = 2.37 \cdot 1.36^x$ A2

(ii) $R^2 = 0.9995695479$

$R^2 = 0.99957$ A1

[3]

(b) $SS_{res} = 0.0135964079$

$SS_{res} = 0.0136$ A2

[2]

(c) $R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$

$0.9995695479 = 1 - \frac{0.0135964079}{SS_{tot}}$ (A1) for substitution

$\frac{0.0135964079}{SS_{tot}} = 0.0004304521$

$SS_{tot} = 31.58634352$

$SS_{tot} = 31.6$ A1

[2]

Chapter 19 Solution

Exercise 75

1. (a) H_0: The data follows a Poisson distribution with mean 2. A1

[1]

(b) 17.1 A1

[1]

(c) 4 A1

[1]

(c) 55.1 A2

[2]

(d) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 7.779$. A1

[2]

2. (a) H_0: The data follows a Binomial distribution with parameters $\mathrm{B}(5, 0.6)$. A1

[1]

(b) (i) 1.024 A1

(ii) 4 A1

[2]

(c) 24.8 A2

[2]

(d) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 9.488$. A1

[2]

3. (a) H_0: The data follows a Normal distribution with parameters $\mathrm{N}(13, 4)$. A1

[1]

(b) (i) 4.01 A1

(ii) 2 A1

[2]

(c) $p\text{-value} = 0.4074438353$ (A1) for correct value

$p\text{-value} = 0.407$ A1

[2]

(d) The null hypothesis is not rejected. A1

As $p\text{-value} > 0.05$. A1

[2]

4. (a) H_0: The data follows a Normal distribution with parameters $\mathrm{N}(3.4, 0.15)$. A1

[1]

(b) 18.2 A1

[1]

(c) 3 A1

[1]

(d) $p\text{-value} = 0.00003755288981$ (A1) for correct value

$p\text{-value} = 0.0000376$ A1

[2]

(e) The null hypothesis is rejected. A1

As $p\text{-value} < 0.01$. A1

[2]

Exercise 76

1.	(a)	(i)	$H_0: \mu = 17$	A1
		(ii)	$H_1: \mu \neq 17$	A1
				[2]
	(b)		$p\text{-value} = 0.0456126344$	(A1) for correct value
			$p\text{-value} = 0.0456$	A1
				[2]
	(c)	The null hypothesis is rejected.		A1
		As $p\text{-value} < 0.05$.		A1
				[2]
2.	(a)	(i)	$H_0: \mu = 28$	A1
		(ii)	$H_1: \mu < 28$	A1
				[2]
	(b)		$p\text{-value} = 0.4115316745$	(A1) for correct value
			$p\text{-value} = 0.412$	A1
				[2]
	(c)	The null hypothesis is not rejected.		A1
		As $p\text{-value} > 0.1$.		A1
				[2]
3.	(a)	(i)	$H_0: \mu = 7$	A1
		(ii)	$H_1: \mu > 7$	A1
				[2]
	(b)	(i)	7.1875	A1
		(ii)	$z = \dfrac{7.1875 - 7}{\dfrac{\sqrt{3}}{\sqrt{16}}}$	(A1) for correct approach
			$z = 0.4330127019$	
			$z = 0.433$	A1
				[3]
	(c)	The null hypothesis is not rejected.		A1
		As $z < 1.645$.		A1
				[2]

4.	(a)	(i)	$H_0: \mu = 100$	A1	
		(ii)	$H_1: \mu \neq 100$	A1	
					[2]
	(b)	(i)	106	A1	
		(ii)	$z = \dfrac{106-100}{\dfrac{\sqrt{28}}{\sqrt{24}}}$	(A1) for correct approach	
			$z = 5.554920599$		
			$z = 5.55$	A1	
					[3]
	(c)	The null hypothesis is rejected.		A1	
		As $\lvert z \rvert > 2.326$.		A1	
					[2]

Exercise 77

1.	(a)	$H_1: \mu_d < 0$	A1	
				[1]
	(b)	p-value $= 0.0121641869$	(A1) for correct value	
		p-value $= 0.0122$	A1	
				[2]
	(c)	-2.99	A1	
				[1]
	(d)	The null hypothesis is not rejected.	A1	
		As p-value > 0.01.	A1	
				[2]
2.	(a)	$H_1: \mu_d > 0$	A1	
				[1]
	(b)	p-value $= 0.1326990196$	(A1) for correct value	
		p-value $= 0.133$	A1	
				[2]
	(c)	1.21	A1	
				[1]
	(d)	The null hypothesis is not rejected.	A1	
		As p-value > 0.05.	A1	
				[2]
3.	(a)	(i) Let $d = x - y$.		
		$H_0: \mu_d = 0$	A1	
		(ii) $H_1: \mu_d < 0$	A1	
				[2]
	(b)	p-value $= 0.2221566281$	(A1) for correct value	
		p-value $= 0.222$	A1	
				[2]
	(c)	The null hypothesis is not rejected.	A1	
		As p-value > 0.1.	A1	
				[2]
	(d)	$\frac{4}{9}$	A1	
				[1]

4. (a) (i) Let $d = x - y$.

$H_0: \mu_d = 0$ A1

(ii) $H_1: \mu_d > 0$ A1

[2]

(b) $p\text{-value} = 0.0197529044$ (A1) for correct value

$p\text{-value} = 0.0198$ A1

[2]

(c) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. A1

[2]

(d) The required probability

$= \left(\frac{5}{6}\right)\left(\frac{4}{5}\right)$ (M1) for valid approach

$= \frac{2}{3}$ A1

[2]

Exercise 78

1.	(a)	(i)	$H_0: \ p = 0.15$	A1
		(ii)	$H_1: \ p < 0.15$	A1
				[2]
	(b)	$P(X \leq 99) = 0.004261876$		(M1) for valid approach
		Thus, the p -value is 0.00426.		A1
				[2]
	(c)	The null hypothesis is rejected.		A1
		As p-value < 0.01.		A1
				[2]
2.	(a)	(i)	$H_0: \ \lambda = 10$	A1
		(ii)	$H_1: \ \lambda > 10$	A1
				[2]
	(b)	$P(X \geq 13) = 1 - P(X \leq 12)$		(M1) for valid approach
		$P(X \geq 13) = 0.208443523$		
		Thus, the p -value is 0.208.		A1
				[2]
	(c)	The null hypothesis is not rejected.		A1
		As p-value > 0.05.		A1
				[2]
3.	(a)	(i)	$H_0: \ p = 0.03$	A1
		(ii)	$H_1: \ p > 0.03$	A1
				[2]
	(b)	(i)	$P(X \geq 7) < 0.05$ and $P(X \geq 6) > 0.05$	R1
			Thus, the least value of n is 7.	A1
		(ii)	6	A1
				[3]

4.	(a)	(i)	$H_0: \lambda = 2.5$	A1	
		(ii)	$H_1: \lambda > 2.5$	A1	
					[2]
	(b)	(i)	$P(X \geq 8) < 0.01$ and $P(X \geq 7) > 0.01$	R1	
			Thus, the least value of n is 8.	A1	
		(ii)	7	A1	
					[3]

Exercise 79

1.	(a)	(i)	$H_0: \lambda = 9$	A1
		(ii)	$H_1: \lambda > 9$	A1
				[2]
	(b)	The required probability		
		$= P(X \geq 14 \mid \lambda = 9)$		(M1) for valid approach
		$= 1 - P(X \leq 13 \mid \lambda = 9)$		
		$= 0.0738507692$		
		$= 0.0739$		A1
				[2]
	(c)	The required probability		
		$= P(X \leq 13 \mid \lambda = 13)$		(M1) for valid approach
		$= 0.5730445628$		
		$= 0.573$		A1
				[2]
	(d)	$0 \leq Y \leq 4$		A2
				[2]
	(e)	$P(Y \leq 6) = 0.0367373531$		(M1) for valid approach
		Thus, the p-value is 0.0367.		A1
				[2]
	(f)	The null hypothesis is not rejected.		A1
		As p-value > 0.01.		A1
				[2]

2.	(a)	(i)	$H_0: \ q = 0.92$	A1
		(ii)	$H_1: \ q > 0.92$	A1
				[2]
	(b)	The required probability		
		$= \mathrm{P}(X \geq 49 \mid q = 0.92)$		(M1) for valid approach
		$= 1 - \mathrm{P}(X \leq 48 \mid q = 0.92)$		
		$= 0.0827120229$		
		$= 0.0827$		A1
				[2]
	(c)	The required probability		
		$= \mathrm{P}(X \leq 48 \mid q = 0.94)$		(M1) for valid approach
		$= 0.8099967419$		
		$= 0.810$		A1
				[2]
	(d)	$Y \geq 7$		A2
				[2]
	(e)	$\mathrm{P}(Y \geq 5) = 1 - \mathrm{P}(Y \leq 4)$		(M1) for valid approach
		$\mathrm{P}(Y \geq 5) = 0.1847367554$		
		Thus, the p-value is 0.185.		A1
				[2]
	(f)	The null hypothesis is not rejected.		A1
		As p-value > 0.05.		A1
				[2]

3. (a) (i) $H_0: \mu = 250$ A1

(ii) $H_1: \mu > 250$ A1

[2]

(b) The required probability

$= P(\bar{X} > 251 | \mu = 250)$ (M1) for valid approach

$= 0.0126736174$

$= 0.0127$ A1

[2]

(c) The required probability

$= P(\bar{X} < 251 | \mu = 250.5)$ (M1) for valid approach

$= 0.868223716$

$= 0.868$ A1

[2]

(d) $\bar{Y} \leq 298$ or $\bar{Y} \geq 302$ A2

[2]

(e) The null hypothesis is rejected. A1

As the sample mean is less than 298. A1

[2]

(f) $\bar{Y} \leq 299$ or $\bar{Y} \geq 301$ A2

[2]

4.	(a)	(i) $H_0: \mu = 100$	A1	
		(ii) $H_1: \mu \neq 100$	A1	
				[2]
	(b)	The required probability		
		$= P(\bar{X} < 99 \cup \bar{X} > 101 \mid \mu = 100)$	(M1) for valid approach	
		$= P(\bar{X} < 99 \mid \mu = 100) + P(\bar{X} > 101 \mid \mu = 100)$		
		$= 0.1336144574$		
		$= 0.134$	A1	
				[2]
	(c)	The required probability		
		$= P(99 < \bar{X} < 101 \mid \mu = 101)$	(M1) for valid approach	
		$= 0.4986500333$		
		$= 0.499$	A1	
				[2]
	(d)	$\bar{Y} \leq 24.1$	A2	
				[2]
	(e)	The null hypothesis is rejected.	A1	
		As the sample mean is less than 24.1.	A1	
				[2]
	(f)	$\bar{Y} \leq 23.7$	A2	
				[2]

Exercise 80

1.	(a)	(i)	$H_0: \rho = 0$	A1
		(ii)	$H_1: \rho < 0$	A1
				[2]
	(b)		$p\text{-value} = 0.0011875732$	(A1) for correct value
			$p\text{-value} = 0.00119$	A1
				[2]
	(c)		The null hypothesis is rejected.	A1
			As $p\text{-value} < 0.1$.	A1
				[2]
	(d)	(i)	$a = -4.242857143$	
			$a = -4.24$	A1
			$b = 99.2$	A1
		(ii)	b represents the expected quiz score of a student who didn't revise.	A1
				[3]
	(e)		The estimated quiz score	
			$= -4.242857143(7) + 99.2$	(A1) for substitution
			$= 69.5$ marks	A1
				[2]
	(f)	(i)	$r = -0.9599392773$	
			$r = -0.960$	A1
		(ii)	$R^2 = 0.921483416$	
			$R^2 = 0.921$	A1
		(iii)	92.1% of the variability of the data is explained by the regression model.	A1
				[3]

2. (a) (i) $H_0:\ \rho = 0$ A1

(ii) $H_1:\ \rho \neq 0$ A1

[2]

(b) $p\text{-value} = 0.0164235283$ (A1) for correct value

$p\text{-value} = 0.0164$ A1

[2]

(c) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. A1

[2]

(d) (i) $a = 0.9371428571$

$a = 0.937$ A1

$b = -5.066666667$

$b = -5.07$ A1

(ii) a represents the average increase in the number of webpages visited when the time for accessing the internet is increased by 1 minute. A1

[3]

(e) The estimated number of webpages visited

$= 0.9371428571(22) - 5.066666667$ (A1) for substitution

$= 15.55047619$

$= 16$ A1

[2]

(f) (i) $r = 0.8934533031$

$r = 0.893$ A1

(ii) $R^2 = 0.7982588049$

$R^2 = 0.798$ A1

(iii) 79.8% of the variability of the data is explained by the regression model. A1

[3]

3. (a) (i) $H_0: \rho = 0$ A1

(ii) $H_1: \rho > 0$ A1

[2]

(b) $p\text{-value} = 0.5186783856$ (A1) for correct value

$p\text{-value} = 0.519$ A1

[2]

(c) The null hypothesis is not rejected. A1

As $p\text{-value} > 0.05$. A1

[2]

(d) This suggestion is not a valid approach. A1

As from the result of the hypothesis test, the two variables are weakly correlated. R1

[2]

(e) (i) $R^2 = 0.0008610792193$

$R^2 = 0.000861$ A1

(ii) 0.0861% of the variability of the data is explained by the regression model. A1

[2]

(f) $SS_{res} = 18822$ A2

[2]

(g) $R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$

$0.9037 = 1 - \frac{18822}{SS_{tot}}$ (A1) for substitution

$\frac{18822}{SS_{tot}} = 0.0963$

$SS_{tot} = 195451.7134$

$SS_{tot} = 195000$ A1

[2]

4. (a) (i) $H_0: \rho = 0$ A1

(ii) $H_1: \rho < 0$ A1

[2]

(b) $p\text{-value} = 0.1808540814$ (A1) for correct value

$p\text{-value} = 0.181$ A1

[2]

(c) The null hypothesis is not rejected. A1

As $p\text{-value} > 0.1$. A1

[2]

(d) This suggestion is not a valid approach. A1

As from the result of the hypothesis test, the two variables are weakly correlated. R1

[2]

(e) (i) $R^2 = 0.2775310835$

$R^2 = 0.278$ A1

(ii) 27.8% of the variability of the data is explained by the regression model. A1

[2]

(f) $SS_{res} = 131.6875$ A2

[2]

(g) $R^2 = 1 - \dfrac{SS_{res}}{SS_{tot}}$

$0.58 = 1 - \dfrac{131.6875}{SS_{tot}}$ (A1) for substitution

$\dfrac{131.6875}{SS_{tot}} = 0.42$

$SS_{tot} = 313.5416667$

$SS_{tot} = 314$ A1

[2]

Chapter 20 Solution

Quick Practice

Part I Solution

(a) (1) $a = -0.7484151116$, $b = 11.46800981$, $c = -57.93589471$,
$d = 450.8750458$
$a = -0.748$, $b = 11.5$, $c = -57.9$, $d = 451$

(2) The estimated number of books sold
$= -0.7484151116(6)^3 + 11.46800981(6)^2 - 57.93589471(6) + 450.8750458$
$= 354.4503666$
$= 354$

(3) This estimation is valid as this is an interpolation.

(b) (1) 346.2

(2) $H_0:\ \mu = 370$

(3) $H_1:\ \mu < 370$

(4) $p\text{-value} = 0.0312739104$
$p\text{-value} = 0.0313$

(5) The null hypothesis is rejected.
As $p\text{-value} < 0.05$.

Part II Solution

(c) (1) $y(n) = 403.9123539 \cdot 0.9791106773^n$

$y(n) = 403.91 \cdot 0.97911^n$

(2) $R^2 = 0.9709573534$

$R^2 = 0.971$

(3) 97.1% of the variability of the data is explained by the regression model.

(d) (1) 6

(2) 1.54

(3) The null hypothesis is not rejected.

As $\chi^2_{calc} < 14.449$.

Part III Solution

(e) (1) Let $\mu_d, d = y - x$ be the mean of the differences of the numbers of books sold by Olga and Nina.

$H_0:\ \mu_d = 0$

(2) $H_1:\ \mu_d > 0$

(3) $p\text{-value} = 0.0583641326$

$p\text{-value} = 0.0584$

(4) The null hypothesis is rejected.

As $p\text{-value} < 0.1$.

(f) (1) The numbers of books sold by the two authors follow a bivariate normal distribution.

(2) $H_0:\ \rho = 0$

(3) $H_1:\ \rho \neq 0$

(4) $p\text{-value} = 0.0111187257$

$p\text{-value} = 0.0111$

(5) The null hypothesis is rejected.

As $p\text{-value} < 0.05$.

(g) (1) 90% confidence interval:

(–0.592, 20.0)

(2) The above result is not consistent with the conclusion of the hypothesis test in (e) as 0 is included in the confidence interval.

Exercise 81

1. (a) $\dfrac{\mathrm{d}v}{\mathrm{d}x} = 4v + 12y$

$\therefore \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) = 4\dfrac{\mathrm{d}y}{\mathrm{d}x} + 12y$ M1

$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4\dfrac{\mathrm{d}y}{\mathrm{d}x} + 12y$ A1

$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - 4\dfrac{\mathrm{d}y}{\mathrm{d}x} - 12y = 0$ AG

[2]

(b) (1) $\mathbf{M} = \begin{pmatrix} 4 & 12 \\ 1 & 0 \end{pmatrix}$ A1

(2) The characteristic polynomial of $\mathbf{M}$

$= \det(\mathbf{M} - \lambda\mathbf{I})$

$= \begin{vmatrix} 4-\lambda & 12 \\ 1 & 0-\lambda \end{vmatrix}$ M1

$= (4-\lambda)(-\lambda) - (12)(1)$

$= \lambda^2 - 4\lambda - 12$ A1

(3) $\lambda_1 = -2$, $\lambda_2 = 6$ A2

(4) $\mathbf{v}_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ A2

(5) $y = Ae^{-2x} + Be^{6x}$ A2

(6) $\frac{dy}{dx} = A(e^{-2x})(-2) + B(e^{6x})(6)$ A1

$\frac{dy}{dx} = -2Ae^{-2x} + 6Be^{6x}$

$\frac{d^2y}{dx^2} = -2Ae^{-2x}(-2) + 6Be^{6x}(6)$ A1

$\frac{d^2y}{dx^2} = 4Ae^{-2x} + 36Be^{6x}$

$\therefore \frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y$

$= (4Ae^{-2x} + 36Be^{6x}) - 4(-2Ae^{-2x} + 6Be^{6x})$ M1

$-12(Ae^{-2x} + Be^{6x})$

$= 4Ae^{-2x} + 36Be^{6x} + 8Ae^{-2x} - 24Be^{6x}$

$-12Ae^{-2x} - 12Be^{6x}$

$= 0$

Thus, the general solution for y satisfies the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = 0$. AG

[12]

(c) (1) The eigenvalues of **M** are the solutions of $\lambda^2 - 4\lambda - 12 = 0$, where -4 and -12 are the coefficients of $\frac{dy}{dx}$ and y in

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = 0$. R2

(2) $\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ A2

[4]

(d) (1) $\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 27y = 0$

$\therefore \lambda^2 - 12\lambda + 27 = 0$ A1

$(\lambda - 3)(\lambda - 9) = 0$

$\lambda = 3$ or $\lambda = 9$ A1

$\therefore y = Ae^{3x} + Be^{9x}$ A1

(2) $3 = Ae^{3(0)} + Be^{9(0)}$

$3 = A + B$ A1

$\frac{dy}{dx} = A(e^{3x})(3) + B(e^{9x})(9)$ M1

$\frac{dy}{dx} = 3Ae^{3x} + 9Be^{9x}$

$15 = 3Ae^{3(0)} + 9Be^{9(0)}$

$15 = 3A + 9B$ A1

By solving this system, $A = 2$ and $B = 1$. A1

$\therefore y = 2e^{3x} + e^{9x}$ A1

[8]

2. (a) (1) -2 A1

(2) $\frac{1}{2}$ A1

(3) The equation of the perpendicular bisector of QR :

$y-7=\frac{1}{2}(x-5)$ M1

$y-7=\frac{1}{2}x-\frac{5}{2}$

$y=\frac{1}{2}x+\frac{9}{2}$ A1

(4) $y=\frac{1}{2}(10)+\frac{9}{2}$ M1

$y=\frac{19}{2}$

Thus, the coordinates of C are $\left(10, \frac{19}{2}\right)$. A1

(5) The required length of the highway

$=\sqrt{(10-5)^2+\left(\frac{19}{2}-7\right)^2}$ M1

$=5.590169944$ A1

$=559$ m A1

[9]

(b) (1) 333 m A1

(2) The total length of the highway

$=559+500+333+2(527)+4(1000)$ M1A1

$=6446$ m

$=6450$ m A1

[4]

(c) (1) 6 A1

(2) 4 A1

[2]

(d) Eulerian circuit does not exist. A1

As not all vertices are of even degree. A1

[2]

(e)	(1)	CJ	A1
	(2)	For any four edges correct	A1
		For all edges correct	A1
		1. Choose CJ of time 10	
		2. Choose HI of time 20	
		3. Choose DE of time 25	
		4. Choose FG of time 25	
		5. Choose AB of time 30	
		6. Choose BH of time 30	
		7. Choose BC of time 35	
		8. Choose AD of time 40	
		9. Choose GH of time 50	
		Thus, the minimum spanning tree is a tree containing the highways CJ, HI, DE, FG, AB, BH, BC, AD and GH.	A1
	(3)	265 s	A1

[5]

(f)	For any five edges correct		A1
	For any ten edges correct		A1
	1.	Choose EF of time 65	
	2.	Choose FG of time 25	
	3.	Choose GH of time 50	
	4.	Choose HB of time 30	
	5.	Choose BC of time 35	
	6.	Choose CB of time 35	
	7.	Choose BA of time 30	
	8.	Choose AD of time 40	
	9.	Choose DA of time 40	
	10.	Choose AG of time 60	
	11.	Choose GH of time 50	
	12.	Choose HI of time 20	
	13.	Choose IJ of time 55	
	14.	Choose JC of time 10	
	15.	Choose CD of time 45	
	16.	Choose DE of time 25	
	Thus, a possible route contains EF, FG, GH, HB, BC, CB, BA, AD, DA, AG, GH, HI, IJ, JC, CD and DE.		A2

[4]

(g)	AD, BC, GH	A3

[3]